Math 1552 Section 17
PracticeTest 2

Formulas:

- Geometric series: the sum of the geometric series $\sum_{n=0}^{\infty} ar^n$ is $\frac{a}{1-r}$.
- Taylor series formulas:
  - General formula for Taylor series centered at $x = c$:

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(c)(x-c)^n$$

  - Maclaurin series for $f(x) = e^x$:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

  - Maclaurin series for $f(x) = \sin x$:

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$

  - Maclaurin series for $f(x) = \cos x$:

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$$

Convergence Tests:

- Integral test: If $a_n = f(n)$ for a suitable function f, then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_{x=1}^{\infty} f(x)dx$ converges.
- p-test: $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$.
- Comparison test: Suppose two series have positive terms and the terms of the first series are eventually smaller than the terms of the second series. Then if the second (larger) series converges, so does the first series. If the first (smaller) series diverges, so does the second series.

- Limit comparison test: Suppose two series have positive terms and the limit of the ratio of their terms is a positive number. Then the behavior of the series is the same: either both converge or both diverge.
- Alternating series test: If the size of the terms in a series decreases to zero and the signs alternate, the series converges.
- Absolute convergence: If you replace the terms of a series with their absolute values and the resulting series converges, then so does the original series.
- Ratio test: if the limit $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$ is less than one, then the series $\sum_{n=1}^{\infty} a_n$ converges. If the limit is greater than one, it diverges. If the limit equals one, the ratio test is inconclusive.
- Root test: if the limit $\lim_{n\to\infty}\sqrt[n]{|a_n|}$ is less than one, then the series $\sum_{n=1}^{\infty} a_n$ converges. If the limit is greater than one, it diverges. If the limit equals one, the root test is inconclusive.

Part I: Conceptual Questions.

1. Give an example of a conditionally convergent series that is not absolutely convergent.

2. Give an example of an alternating series that does not converge.

Part II: Finding Sums.

3. Find the sum of the series $\frac{1}{5^2} - \frac{1}{5^4} + \frac{1}{5^6} - \frac{1}{5^8} + \ldots$ if it converges.

4. Integrate both sides of the geometric series formula to find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n2^n}$.

Part III: Determining Convergence.

5. Does the series $\sum_{n=0}^{\infty} \pi^{-n}$ converge? Why or why not?

6. Does the series $\sum_{n=1}^{\infty} \frac{n^n}{n^{n^n}}$ converge? Why or why not?

7. Does the series $\sum_{n=1}^{\infty} \frac{(2n)!}{(2n+1)!}$ converge? Why or why not?

Part IV: Power Series.

8. Calculate the Taylor series for $f(x) = xe^x$ centered at $x = 1$.

9. Find the center and radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{n(\ln n)^2}$. Does it converge at the endpoints? Why or why not?

10. Suppose $f(x)$ is infinitely many times differentiable for all x and $f^{(n)}(0) = 1$ for all n. What is $f(x)$?