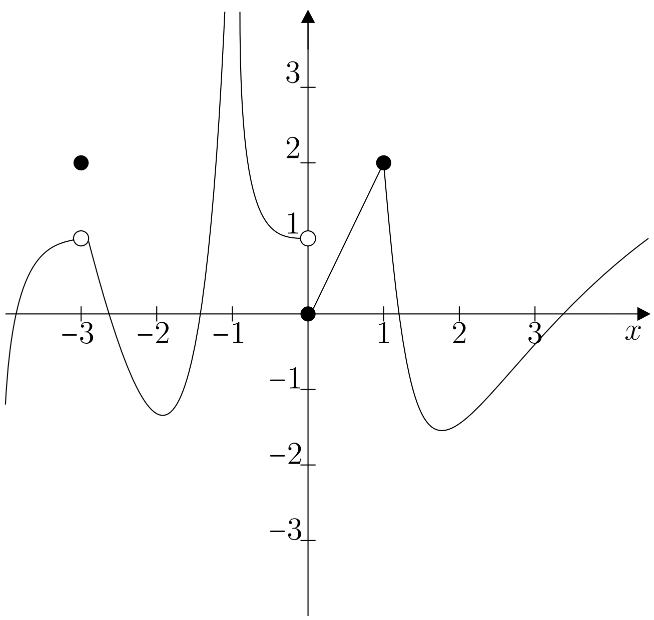
**Math 1550 Section 20** **Name:** ____________________________
**Final Exam**
**Friday, December 7, 2012**

**Instructions:** You have 120 minutes to complete the exam. Be sure to work both sides of each page! There are 18 problems and two bonus problems. Each problem is worth the same amount. The maximum possible score is 111 out of 100. You may use your calculator. Good luck!

1. Consider the function $f(x)$ whose graph is shown below:




a) At which points does the limit of $f(x)$ not exist? Why?

b) Does $f(x)$ have asymptotes? If so, where are they?

c) At which points is $f(x)$ discontinuous? Why?

d) At which points is $f(x)$ non-differentiable? Why?

e) Extend the graph to the right in such a way that $f(x)$ has a removable discontinuity at $x = 6$ and a horizontal asymptote at $y = 2$. (Draw this in with your pencil on the above graph.)

2. The Squeeze Theorem uses two functions $g(x)$ and $h(x)$, whose limits are "easy to find," to "bracket" or "squeeze" a function $f(x)$, whose limit is "hard to find." If $f(x) = x^2 \cos \frac{1}{x}$, and if you want to find the limit of $f(x)$ as $x$ approaches zero, which of the following pairs of functions will work for $g(x)$ and $h(x)$? Circle all pairs that will work, and explain your choices.

a) $g(x) = -x, h(x) = x$.

b) $g(x) = -|x|, h(x) = |x|$.

c) $g(x) = -x^2, h(x) = x^2$.

d) $g(x) = -x^4, h(x) = x^4$.

3. Find a function $f(x)$ such that $\lim_{x \to -\infty} f(x) = 0$, $\lim_{x \to 0^-} f(x) = -\infty$, $\lim_{x \to 0^+} f(x) = \infty$, and $\lim_{x \to \infty} f(x) = 0$. Draw this function.

4. State the definition of the derivative of $f(x)$.

5. Find the equation of the tangent line to $f(x) = 2x^2$ at the point $(2, 8)$.

6. Find the derivative of $f(x) = \sin(e^{3x^2})$ using the chain rule.

7. A 10-foot ladder is leaning against a wall. At $t = 0$ seconds, the base of the ladder is 4 feet from the wall, and begins to slide away from the wall at 2 feet per second. How fast is the top of the ladder moving down the wall at $t = 2$ seconds?

8. Water flows into a cone-shaped cup at 2 cubic centimeters per second. The top of the cup has a radius of 5 centimeters, and the height of the cup is 10 centimeters. How fast is the level of the water in the cup rising when it is 1 centimeter from the top?

9. Let $f(x) = x^3 - x$. Find the critical points, the maximum and minimum values, the points of inflection, and the intervals on which $f(x)$ is increasing, decreasing, concave up, and concave down, respectively. Sketch the graph of $f(x)$.

10. Prove that the rectangle with largest area having a perimeter of 100 feet is a square with 25-foot sides.

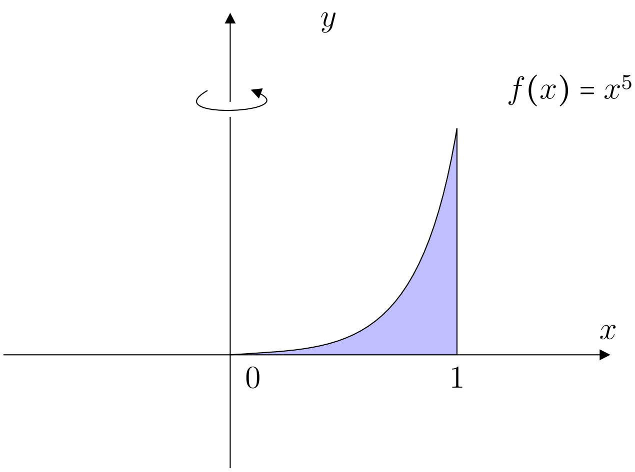
11. State the Fundamental Theorem of Calculus.

12. What is $\int_0^2 (x^3 - 3x^2 + 2x + 1)dx$?

13. Find a function $f(x)$ such that $\int_a^b f(x)dx = f(b) - f(a)$. (Hint: I *do* mean $f$, not $F$, on the right-hand side.)

14. Calculate $\int x \sin^2(x^2) \cos(x^2) dx$.

15. Calculate the volume of the solid obtained by revolving the region between the graph of $f(x) = x^5$ and the $x$-axis from 0 to 1 around the $y$-axis, using the method of cylindrical shells.




16. What is the average value of the function $f(x) = x^2$ from $x = 0$ to $x = 2$?

17. Suppose that 12 joules of work is required to stretch a spring 20 centimeters beyond its natural length. What is the value of the spring constant in newtons per meter?

18. What is the arc length of the graph of the function $f(x) = x^{\frac{3}{2}}$ from $x = 0$ to $x = 1$?

BONUS 1: What is the surface area of the surface of revolution given by rotating the graph of the function $f(x) = \sqrt{100 - x^2}$ from $x = -10$ to $x = 10$ about the $x$-axis?

BONUS 2: Find the following:

a) A nonzero function equal to the square of its derivative.

b) A nonzero function equal to one-eighth its fourth derivative.

c) A nonzero function equal to its second derivative divided by its first derivative.

Hint: for the last two, think about trigonometric functions. Problems like these arise in the study of *differential equations.*