Math 1552 Section 16
Practice Final Exam

Part I: Techniques of Integration. Please compute the following integrals.

1. $\int x \ln x dx$

2. $\int \sin^2\theta \cos\theta d\theta$

3. $\int \frac{1}{(x+4)(x-1)} dx$

4. $\int_0^\infty xe^{-x^2} dx$

Part II: Parametric Equations.

5. Consider the parametric equations $x(t) = 1+t$, $y(t) = t^2$, for $-1 \le t \le 1$. Sketch the parametric curve, including arrows to show motion along the curve as t increases. Solve for y in terms of x.

6. Use the arc length formula $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$ to compute the arc length of the parametric curve $x(t) = \cos t$, $y(t) = \sin t$ from $t = 0$ to $t = \frac{\pi}{2}$.

Part III: Conic Sections.

7. Put the equation $x^2 + y^2 + 2x - 4y = 4$ into standard form by completing the square. Tell what kind of conic section it is, and make an accurate sketch.

8. Find the vertices and asymptotes of the hyperbola $x^2 - y^2 = 1$. Make an accurate sketch.

Part IV: Sequences and Series.

9. Find the sum of the series $\sum_{n=0}^{\infty} \frac{1}{4^n}$.

10. Match each series to the best test for determining whether or not it converges.

$\sum_{n=1}^{\infty} \frac{n!}{n^n}$	Comparison Test
$\sum_{n=1}^{\infty} \frac{n^2+1}{n^4}$	Limit Comparison Test
$\sum_{n=1}^{\infty} (-1)^n \frac{n^4}{n^2+1}$	Root Test
$\sum_{n=1}^{\infty} \frac{1}{n \ln n}$	Alternating Series Test
$\sum_{n=1}^{\infty} \frac{n^2}{n^4+1}$	Divergence Test
$\sum_{n=1}^{\infty} \frac{3^n}{2^{2n}}$	Integral Test
$\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{n^4}$	Ratio Test

11. Use the integral test to determine whether or not the series $\sum_{n=1}^{\infty} ne^{-n}$ converges.

12. Write the series $\frac{1}{3} + \frac{4}{9} + \frac{9}{27} + \frac{16}{81} + \frac{25}{243} + ...$ in summation notation. Does it converge? How do you know?

13. Find the first five terms of the Taylor series for $f(x) = \sin x$ centered at $x = \frac{\pi}{2}$.

14. Find the first four terms of the MacLaurin series for $f(x) = \ln(1 + x)$.

Part V: Vector Algebra.

15. Let $\mathbf{v} = \langle 1, 2 \rangle$ and $\mathbf{w} = \langle -2, 1 \rangle$ be vectors in the plane. Compute $\mathbf{v} + \mathbf{w}$ and $\mathbf{v} - \mathbf{w}$. Make accurate sketches of $\mathbf{v}$, $\mathbf{w}$, $\mathbf{v} + \mathbf{w}$, and $\mathbf{v} - \mathbf{w}$. Compute the lengths of all four vectors. Find unit vectors pointing in the directions of $\mathbf{v}$ and $\mathbf{w}$.

16. Find the dot product of the vectors $\mathbf{v} = \langle 1, 1, 1 \rangle$ and $\mathbf{w} = \langle 2, -1, -1 \rangle$. What is the angle between $\mathbf{v}$ and $\mathbf{w}$? What is the component of $\mathbf{w}$ parallel to $\mathbf{v}$?

17. Find two different unit vectors orthogonal to both of the vectors $\mathbf{v} = \langle 2, 0, 0 \rangle$ and $\mathbf{w} = \langle 0, 2, 0 \rangle$.

Part VI: Three-Dimensional Geometry.

18. Find an equation for the plane containing the point $(0,1,0)$, with normal vector $\langle 1,0,0\rangle$.

19. Sketch and describe the surfaces defined by the equations $x^2+y^2=1$, $x^2-y^2=1$, and $x^2+y^2=z$.

Part VII: Partial Derivatives.

20. Let $f(x,y)=x^2y+2y^3$. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. Show that $\frac{\partial^2 f}{\partial y \partial x}=\frac{\partial^2 f}{\partial x \partial y}$.