

There are eight questions plus one bonus question. Be sure to show your work. You may use your calculator. Good luck!

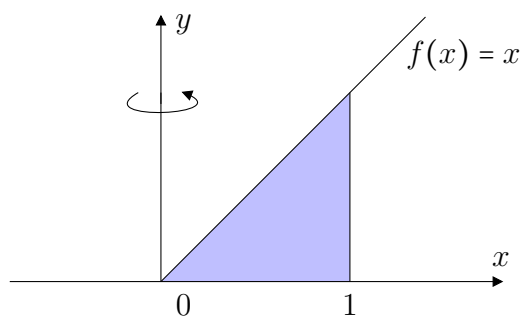
1. Calculate  $\int_0^1 100x^{99} dx$ .

2. Calculate  $\int_{-R}^R \sqrt{R^2 - x^2} dx$ . (Hint:  $x^2 + y^2 = R^2$  is the equation of a circle centered at the origin with radius  $R$ .)

3. Calculate  $\int_1^{e^2} \frac{d}{dx} \ln x dx$ .

4. Calculate  $\frac{d}{dx} \int_0^x \cos t dt$ .

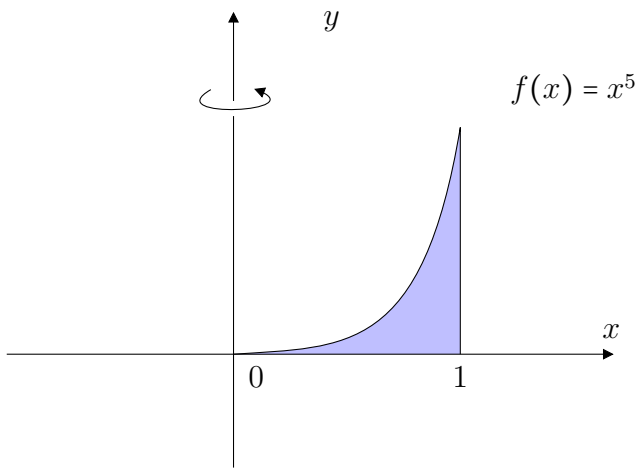
5. Compute the volume of the solid of revolution given by revolving the region between the graph of  $f(x) = x$  and the  $x$ -axis from  $x = 0$  to  $x = 1$  around the  $y$ -axis, using the method of washers.



6. Calculate  $\int \frac{1}{\sin x \cos x} dx$ . (Hint: First rewrite the integrand in terms of  $\sec x$  and  $\tan x$ .)

7. Calculate  $\int \frac{xe^{x^2}}{1+e^{2x^2}} dx$  (Hint:  $\frac{d}{du} \tan^{-1}(u) = \frac{1}{1+u^2}$ ).

8. Calculate the volume of the solid obtained by revolving the region between the graph of  $f(x) = x^5$  and the  $x$ -axis from 0 to 1 around the  $y$ -axis, using the method of cylindrical shells.



BONUS: Calculate  $\int_0^{\infty} e^{-x} dx$  by replacing  $\infty$  with a variable  $R$  in the upper limit of integration, and taking the limit of the resulting expression as  $R \rightarrow \infty$ . This is called an *improper integral*.