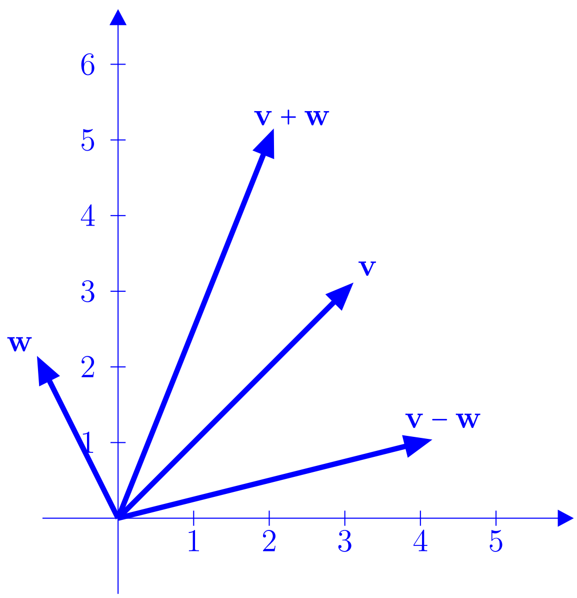
Math 1552 Section 16 **Name:** ____________________
Test 3
Monday, November 25, 2013

Part I: Vector Addition and Subtraction.

1. Let $\mathbf{v} = \langle 3, 3\rangle$ and $\mathbf{w} = \langle -1, 2\rangle$ be vectors in the plane. Compute $\mathbf{v} + \mathbf{w}$ and $\mathbf{v} - \mathbf{w}$. Make accurate sketches of $\mathbf{v}$, $\mathbf{w}$, $\mathbf{v} + \mathbf{w}$, and $\mathbf{v} - \mathbf{w}$. Compute the lengths of all four vectors. Find unit vectors pointing in the directions of $\mathbf{v}$ and $\mathbf{w}$.

   Solution: $\mathbf{v} + \mathbf{w} = \langle \mathbf{2}, \mathbf{5}\rangle$, $\mathbf{v} - \mathbf{w} = \langle \mathbf{4}, \mathbf{1}\rangle$.

   Sketches:



   $\|\mathbf{v}\| = \mathbf{2\sqrt{3}}$, $\|\mathbf{w}\| = \mathbf{\sqrt{5}}$, $\|\mathbf{v} + \mathbf{w}\| = \mathbf{\sqrt{29}}$, $\|\mathbf{v} - \mathbf{w}\| = \mathbf{\sqrt{17}}$.

   Unit vectors: $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \left\langle \dfrac{\mathbf{1}}{\sqrt{\mathbf{2}}}, \dfrac{\mathbf{1}}{\sqrt{\mathbf{2}}} \right\rangle$, $\dfrac{\mathbf{w}}{\|\mathbf{w}\|} = \left\langle \dfrac{\mathbf{-1}}{\sqrt{\mathbf{5}}}, \dfrac{\mathbf{2}}{\sqrt{\mathbf{5}}} \right\rangle$.

2. Write the vector $\mathbf{u} = \langle 5, 5\rangle$ as $a\mathbf{v} + b\mathbf{w}$ for the vectors $\mathbf{v}$ and $\mathbf{w}$ in problem 1, where a and b are real numbers.

   Solution: $a = \dfrac{\mathbf{5}}{\mathbf{3}}$, $b = \mathbf{0}$.

Part II: Dot and Cross Product.

3. Find the dot product of the vectors $\mathbf{v} = \langle 1, 1, 1\rangle$ and $\mathbf{w} = \langle 0, 5, 2\rangle$. What is the angle between $\mathbf{v}$ and $\mathbf{w}$? What is the component of $\mathbf{w}$ parallel to $\mathbf{v}$? What is the component of $\mathbf{v}$ parallel to $\mathbf{w}$?

Solution: $\mathbf{v} \cdot \mathbf{w} = \mathbf{7}$, $\theta = \cos^{-1} \dfrac{\mathbf{7}}{\sqrt{\mathbf{3}}\sqrt{\mathbf{29}}}$, $\text{comp}_{\mathbf{v}} \mathbf{w} = \dfrac{\mathbf{7}}{\sqrt{\mathbf{29}}}$, $\text{comp}_{\mathbf{w}} \mathbf{v} = \dfrac{\mathbf{7}}{\sqrt{\mathbf{3}}}$.

4. What is the dot product of two vectors of lengths 2 and 5 with an angle of $\pi/4$ between them? Can two vectors of length 3 have a dot product of 8? What about 9? What about 10? Explain.

   Solution: $\mathbf{5\sqrt{2}}$, **yes, yes, no.**

5. Find two different unit vectors orthogonal to both of the vectors $\mathbf{v} = \langle 1, 1, 1 \rangle$ and $\mathbf{w} = \langle 1, -1, -1 \rangle$.

   Solution: Solution: compute the cross products $\mathbf{v} \times \mathbf{w}$ and $\mathbf{w} \times \mathbf{v}$, then divide by the lengths. This gives

6. Find the scalar triple product of the vectors $\langle 1, 0, 0 \rangle$, $\langle 0, 2, 0 \rangle$ and $\langle 0, 0, 3 \rangle$.

   Solution: It's the volume of the parallelepiped defined by the three vectors, which is just 6.

Part III: Lines and Planes in Three-Dimensional Space.

7. Is the line through $(-4,-6,1)$ and $(-2,0,-3)$ parallel to the line through $(13,19,9)$ and $(8,4,19)$?

   Solution: Yes, the differences between x, y, and z values are in the same proportions.

8. Find an equation for the plane containing the point $(1,1,1)$, with normal vector $\langle 1,1,1\rangle$. Find an equation for the plane containing the three points $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$.

   Solution: $x + y + z = 3$,

Part IV: Quadric Surfaces.

9. Sketch and describe the surfaces defined by the equations $x^2+y^2=1$, $x^2-y^2=1$, and $x^2+y^2=z$.

   Solution:

10. Consider the surface S defined by the equation $x^2-y^2=z$. Describe the traces of S given by setting x, y, or z constant. What kind of curves are these traces?

BONUS PROBLEMS

11. Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in three-dimensional space. Tell which of the following expressions are meaningful, and what their meaning is.

    a) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$

    b) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$

    c) $\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$

    d) $\mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})$

    e) $\mathbf{u} \times (\mathbf{u} \times \mathbf{u})$

    f) $(\mathbf{u} \times \mathbf{v}) \times (\mathbf{v} \times \mathbf{w})$

    Solution:

12. Let S_1 and S_2 be two planes in three-dimensional space. Suppose that S_1 contains the point $(0,0,0)$, S_2 contains the point $(2,0,0)$, and the two planes share the normal vector $\mathbf{n} = \langle 1,0,1 \rangle$. What is the distance between the two planes?

    Solution: