Math 1550 Section 20
Practice Test 4
Monday November 12, 2012

1. Calculate $\int_0^1 x^{14} dx$.

Solution: $\mathbf{\frac{1}{15}}$. Easy!

2. Calculate $\int_0^1 0\sqrt{100 - x^2}\, dx$. (Hint: the equation of a circle centered at the origin with radius 10 is $x^2 + y^2 = 100$.)

Solution: The integral is one quarter of the area of a circle of radius 10, which is $\mathbf{25}\pi$.

3. Calculate $\int_1^e \frac{d}{dx} \ln x dx$. (Hint: Use the Fundamental Theorem of Calculus and the fact that $\ln x$ is an antiderivative of its derivative.)

Solution: Since the antiderivative is $\ln x$, the FTC says

$$\int_1^e \frac{d}{dx} \ln x dx = \ln e - \ln 1 = 1 - 0 = \mathbf{1}.$$

4. Find a nonzero constant a such that $\int_0^a \sin(x) dx = 0$. (Hint: Think about areas canceling.)

Solution: The integral is

$$\int_0^a \sin x dx = -\cos x\Big|_0^a = -\cos a + \cos 0 = 1 - \cos a,$$

so we need to find some number a such that $\cos a = 1$. $\mathbf{a} = \mathbf{2}\pi$ will do.

5. Calculate $f(x) = \int_0^x \left(\int_0^y \left(\int_0^z dt \right) dz \right) dy$ by doing the innermost integral first, then the next integral, and so on.

Solution: The innermost integral is

$$\int_0^z dt = t\Big|_0^z = z,$$

so the middle integral is

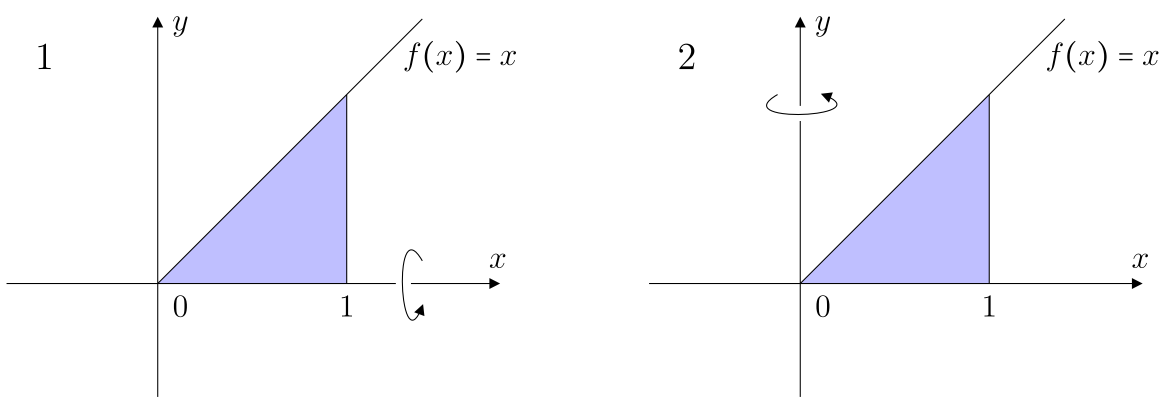
$$\int_0^y z dz = \frac{1}{2}z^2\Big|_0^y = \frac{1}{2}y^2,$$

so the outer integral is

$$\int_0^x \frac{1}{2}y^2 dy = \frac{1}{6}y^3\Big|_0^x = \mathbf{\frac{1}{6}x^3}.$$

If you had n nested integrals, the answer would be $\frac{x^n}{n!}$.

6. Consider two solids of revolution. The first comes from revolving the region between the graph of $f(x) = x$ and the x-axis from $x = 0$ to $x = 1$ around the x-axis, and the second comes from revolving the same region around the y-axis. What is the sum of the two volumes?



Solution: The two solids fit together to form a cylinder of radius 1 and height 1, with volume π.

7. Calculate $\int x^{-1}dx$.

Solution: $\ln \mathbf{x} + \mathbf{C}$.

8. Calculate $\int \frac{\sin(x)}{\cos^2(x)}dx$. (Hint: Use integration by substitution.)

Solution: Let $u = \cos x$. Then $du = -\sin x dx$, so the integral is

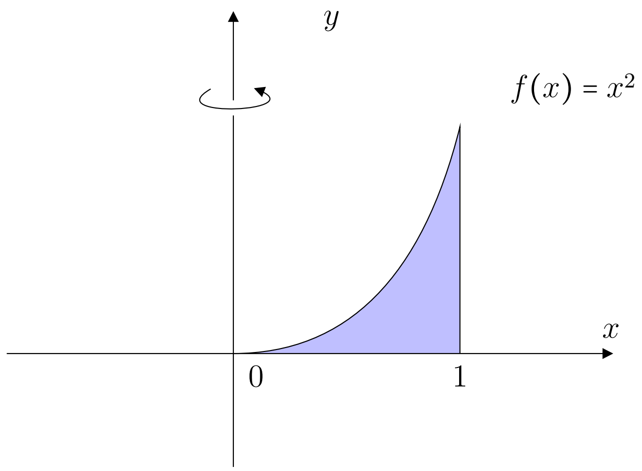
$$\int \frac{\sin(x)}{\cos^2(x)}dx = -\int \frac{1}{u^2}du = -\frac{u^{-1}}{-1} + C = u^{-1} + C = (\cos x)^{-1} + C = \sec \mathbf{x} + \mathbf{C}.$$

9. Calculate $\int \frac{xe^{x^2}}{1+e^{2x^2}}dx \left(\text{Hint: } \frac{d}{du}\tan^{-1}(u) = \frac{1}{1+u^2}\right)$.

Solution: Let $u = e^{x^2}$. Then $du = 2xe^{x^2}dx$, so the integral is

$$\int \frac{xe^{x^2}}{1+e^{2x^2}}dx = \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\tan^{-1} u + C = \frac{\mathbf{1}}{\mathbf{2}}\tan^{\mathbf{-1}} \mathbf{e}^{\mathbf{x^2}} + \mathbf{C}.$$

10. Calculate the volume of the solid obtained by revolving the region between the graph of $f(x) = x^2$ and the x-axis from 0 to 1 around the y-axis, using the method of cylindrical shells.



Solution: The radius of a representative shell is x, and the height is $f(x) = x^2$. So the volume integral is

$$V = \int_0^1 (2\pi)(x)(x^2)dx = 2\pi \int_0^1 x^3 dx = 2\pi \frac{x^4}{4}\Big|_0^1 = \frac{\pi}{\mathbf{2}}.$$