Math 1552 Section 16 **Name:** ______________________________
Test 3
Monday, November 25, 2013

Instructions: You have 110 minutes to take the test. There are 10 problems and two bonus problems. Each problem is worth 10 points. The maximum score is 120 out of 100. You may find the bonus problems difficult, so you should consider doing all you can on the other problems before attempting them. There is partial credit, so show all your work. You may use your calculator, but you must justify all your steps mathematically. Be sure to work both sides of each page. Good luck!

Part I: Vector Addition and Subtraction.

1. Let $\mathbf{v} = \langle 3, 3 \rangle$ and $\mathbf{w} = \langle -1, 2 \rangle$ be vectors in the plane. Compute $\mathbf{v} + \mathbf{w}$ and $\mathbf{v} - \mathbf{w}$. Make accurate sketches of $\mathbf{v}$, $\mathbf{w}$, $\mathbf{v} + \mathbf{w}$, and $\mathbf{v} - \mathbf{w}$. Compute the lengths of all four vectors. Find unit vectors pointing in the directions of $\mathbf{v}$ and $\mathbf{w}$.

2. Write the vector $\mathbf{u} = \langle 5, 5 \rangle$ as $a\mathbf{v} + b\mathbf{w}$ for the vectors $\mathbf{v}$ and $\mathbf{w}$ in problem 1, where a and b are real numbers.

Part II: Dot and Cross Product.

3. Find the dot product of the vectors $\mathbf{v} = \langle 1, 1, 1 \rangle$ and $\mathbf{w} = \langle 0, 5, 2 \rangle$. What is the angle between $\mathbf{v}$ and $\mathbf{w}$? What is the component of $\mathbf{w}$ parallel to $\mathbf{v}$? What is the component of $\mathbf{v}$ parallel to $\mathbf{w}$?

4. What is the dot product of two vectors of lengths 2 and 5 with an angle of $\pi/4$ between them? Can two vectors of length 3 have a dot product of 8? What about 9? What about 10? Explain.

5. Find two different unit vectors orthogonal to both of the vectors $\mathbf{v} = \langle 1, 1, 1 \rangle$ and $\mathbf{w} = \langle 1, -1, -1 \rangle$.

6. Find the scalar triple product of the vectors $\langle 1, 0, 0 \rangle$, $\langle 0, 2, 0 \rangle$ and $\langle 0, 0, 3 \rangle$.

Part III: Lines and Planes in Three-Dimensional Space.

7. Is the line through $(-4, -6, 1)$ and $(-2, 0, -3)$ parallel to the line through $(13, 19, 9)$ and $(8, 4, 19)$?

8. Find an equation for the plane containing the point $(1, 1, 1)$, with normal vector $\langle 1, 1, 1 \rangle$. Find an equation for the plane containing the three points $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$.

Part IV: Quadric Surfaces.

9. Sketch and describe the surfaces defined by the equations $x^2+y^2=1$, $x^2-y^2=1$, and $x^2+y^2=z$.

10. Consider the surface S defined by the equation $x^2-y^2=z$. Describe the traces of S given by setting x, y, or z constant. What kind of curves are these traces?

BONUS PROBLEMS

11. Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in three-dimensional space. Tell which of the following expressions are meaningful, and what their meaning is.

    a) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$

    a) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$

    a) $\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$

    a) $\mathbf{u} \cdot (\mathbf{v} \cdot \mathbf{w})$

    a) $\mathbf{u} \times (\mathbf{u} \times \mathbf{u})$

    a) $(\mathbf{u} \times \mathbf{v}) \times (\mathbf{v} \times \mathbf{w})$

12. Let S_1 and S_2 be two planes in three-dimensional space. Suppose that S_1 contains the point $(0,0,0)$, S_2 contains the point $(2,0,0)$, and the two planes share the normal vector $\mathbf{n} = \langle 1,0,1 \rangle$. What is the distance between the two planes?