Math 1552 Section 16 **Name:** ______________________________
Test 2, Wednesday, October 30, 2011

Instructions: You have 110 minutes to take the test. There are 10 problems and two bonus problems. Each problem is worth 10 points. The maximum score is 120 out of 100. You may find the bonus problems difficult, so you should consider doing all you can on the other problems before attempting them. There is partial credit, so show all your work. You may use your calculator, but you must justify all your steps mathematically. Be sure to work both sides of each page. Good luck!

Formulas:

- Parabola with focus at $(0,p)$ and directrix $y=-p$: $y=\frac{1}{4p}x^2$.
- Ellipse centered at $(0,0)$; major axis $2a$ along x-axis; minor axis $2b$ along y-axis: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
- Hyperbola centered at $(0,0)$; vertices $(\pm a,0)$ asymptotes $y=\pm(b/a)x$: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
- Geometric series: the sum of the geometric series $\sum_{n=0}^{\infty} ar^n$ is $\frac{a}{1-r}$ if $|r|<1$. Otherwise the series diverges.

Convergence Tests:

- Integral test: If $a_n=f(n)$ for a suitable function f, then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_{x=1}^{\infty} f(x)dx$ converges.
- p-test: $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges if and only if $p>1$.
- Comparison test: Suppose two series have positive terms and the terms of the first series are eventually smaller than the terms of the second series. Then if the second (larger) series converges, so does the first series. If the first (smaller) series diverges, so does the second series.
- Limit comparison test: Suppose two series have positive terms and the limit of the ratio of their terms is a positive number. Then the behavior of the series is the same: either both converge or both diverge.
- Alternating series test: If the size of the terms in a series decreases to zero and the signs alternate, the series converges.
- Absolute convergence: If you replace the terms of a series with their absolute values and the resulting series converges, then so does the original series.
- Ratio test: if the limit $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$ is less than one, then the series $\sum_{n=1}^{\infty} a_n$ converges. If the limit is greater than one, it diverges. If the limit equals one, the ratio test is inconclusive.
- Root test: if the limit $\lim_{n\to\infty}\sqrt[n]{|a_n|}$ is less than one, then the series $\sum_{n=1}^{\infty} a_n$ converges. If the limit is greater than one, it diverges. If the limit equals one, the root test is inconclusive.

Part I: Conic Sections.

1. Put the equation $x^2 + 3y^2 + 2x - 12y + 10 = 0$ into standard form. Tell what kind of conic section it is. If it is a parabola, give the focus and directrix. If it is an ellipse, tell where it is centered, and give the major and minor axes and foci. If it is a hyperbola, tell where it is centered, and give the vertices and asymptotes. Make an accurate sketch.

2. Give an equation for a parabola with focus $(1, 0)$ and directrix $x = -1$. Make an accurate sketch.

3. Find the vertices and asymptotes of the hyperbola $x^2 - 4y^2 = 16$. Make an accurate sketch.

Part II: Sequences and Series.

3. Use the ratio test to determine whether the series $\sum_{n=2}^{\infty} \frac{1}{\ln(n!)}$ converges.

4. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n}$.

5. Use the integral test to determine whether or not the series $\sum_{n=1}^{\infty} ne^{-n}$ converges.

6. Is the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^5 6^n}{n!}$ absolutely convergent, conditionally convergent, or divergent? Explain your reasoning.

7. Match each series to the best test for determining whether or not it converges.

$\sum_{n=1}^{\infty}\frac{n!}{n^n}$	Comparison Test
$\sum_{n=1}^{\infty}\frac{n^2+1}{n^4}$	Limit Comparison Test
$\sum_{n=1}^{\infty}(-1)^n\frac{n^4}{n^2+1}$	Root Test
$\sum_{n=1}^{\infty}\frac{1}{n\ln n}$	Alternating Series Test
$\sum_{n=1}^{\infty}\frac{n^2}{n^4+1}$	Divergence Test
$\sum_{n=1}^{\infty}\frac{3^n}{2^{2n}}$	Integral Test
$\sum_{n=1}^{\infty}(-1)^n\frac{n^2+1}{n^4}$	Ratio Test

8. Does the series $\sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}$ converge? Why or why not?

9. Use the root test to show that the series $\sum_{n=1}^{\infty} \frac{2^n}{n^n}$ converges.

10. Write the series $\frac{1}{2} + \frac{4}{4} + \frac{9}{8} + \frac{16}{16} + \frac{25}{32} + ...$ in summation notation. Does it converge? How do you know?

11. BONUS 1: Find the sum of the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$.

12. BONUS 2: Does the series $\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{(n!)}}$ converge?