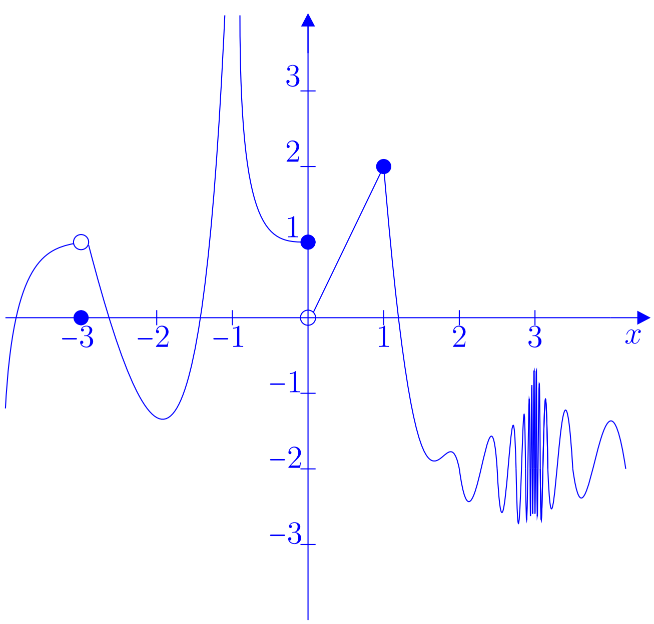
Math 1550 Section 20
Practice Test 1 Solutions

Solutions are in red. **Answers** are in blue.

1. Sketch a function $f(x)$ with the following properties:

- $\lim_{x\to -3} f(x) = 1$, but $f(x)$ is not continuous at $x = -3$.
- $\lim_{x\to -1} f(x) = \infty$.
- $\lim_{x\to 0^-} f(x) = 1$, but $\lim_{x\to 0^+} f(x) = 0$
- $f(x)$ is continuous but non-differentiable at $x = 1$.
- $\lim_{x\to 3} f(x)$ does not exist.

Solution: The function from Quiz 1 will do:



2. Let $f(x) = \dfrac{x^2 - x - 2}{(x-2)(x+1)}$.

a) Find $\lim_{x \to -1} f(x)$, or state that the limit does not exist.

Solution: The numerator factors into $(x-2)(x+1)$, which is the same as the denominator. Therefore all the factors cancel, and

$$\lim_{\mathbf{x} \to -\mathbf{1}} \mathbf{f}(\mathbf{x}) = \lim_{\mathbf{x} \to -\mathbf{1}} \mathbf{1} = \mathbf{1}.$$

b) Find $\lim_{x \to 2} f(x)$, or state that the limit does not exist.

Solution: By the same reasoning,

$$\lim_{\mathbf{x} \to \mathbf{2}} \mathbf{f}(\mathbf{x}) = \lim_{\mathbf{x} \to -\mathbf{1}} \mathbf{1} = \mathbf{1}.$$

3. Use the squeeze theorem to show that $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$.

Solution: The functions $\mathbf{g}(\mathbf{x}) = \cos \mathbf{x}$ and $\mathbf{h}(\mathbf{x}) = \mathbf{1}$ will work as the lower and upper bounds.

4. Use the definition of the derivative of $f(x)$ to find $\dfrac{d}{dx} x^2$. (Don't just write the answer!)

Solution:

$$\begin{aligned} \frac{d}{dx} x^2 &= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h} \\ &= \lim_{h \to 0} \frac{2hx + h^2}{h} \\ &= \lim_{h \to 0} (2x + h) \\ &= \mathbf{2x}. \end{aligned}$$

5. Let $f(x) = (x-1)^3$. Find $f'(x)$ and $f'(1)$.

Solution: Easiest to use chain rule:

$$\mathbf{f'(x) = 3(x-1)^2} \qquad \text{so} \qquad \mathbf{f'(1) = 0}.$$

Could also multiply out the binomial: $(x-1)^3 = x^3 - 3x^2 + 3x - 1$, then differentiate term-by-term.

6. Find the equation of the tangent line to $f(x) = \sqrt{x}$ at the point $(64, 8)$.

Solution: Use the point-slope formula:

$$y - y_1 = m(x - x_1),$$

where m is the slope of the tangent line, $x_1 = 64$, and $y_1 = 8$. We find m by taking the derivative at $x = 64$ ::

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}, \qquad \text{so} \qquad f'(64) = m = \frac{1}{2\sqrt{64}} = \frac{1}{16}.$$

Substituting these values into the point-slope formula, we get the equation $y - 8 = \frac{1}{16}(x - 64)$, or

$$\mathbf{y = \frac{1}{16}x + 4}.$$

7. Let $f(x) = x^2 e^x$. Find $f'(1)$.

Solution: Use the product rule:

$$f'(x) = 2xe^x + x^2 e^x = (x^2 + 2x)e^x,$$

So

$$\mathbf{f'(1) = 3e}.$$

8. Let $f(x) = \sqrt{x}$. For $\varepsilon = \frac{1}{10}$, find $\delta > 0$ such that $|f(x) - 3| < \varepsilon$ whenever $|x - 9| < \delta$.

Solution: Begin with the inequality $|\sqrt{x} - 3| < \frac{1}{10}$ and work as follows:

$$-\frac{1}{10} < \sqrt{x} - 3 < \frac{1}{10},$$

$$\frac{29}{10} < \sqrt{x} < \frac{31}{10}, \qquad \text{(add 3)}$$

$$\frac{841}{100} < x < \frac{961}{100}. \qquad \text{(square)}$$

Thus, x must be in the interval $(\frac{841}{100}, \frac{961}{100})$. The left endpoint is a distance of $\frac{59}{100}$ from 9, and the right endpoint is a distance of $\frac{61}{100}$ from 9. Therefore, we must choose

$$\delta < \mathbf{\frac{59}{100}}.$$