Math 1552 Section 16 **Name:** ____________________________

Test 1, October 2, 2013

Instructions: You have 110 minutes to take the test. There are 10 problems and one bonus problem. Each problem is worth 10 points. The maximum score is 110 out of 100. There is partial credit, so show all your work. You may use your calculator, but you must justify all your steps mathematically. Be sure to work both sides of each page. Good luck!

Formulas:

- Integration by parts: $\int u\,dv = uv - \int v\,du.$
- Derivatives of inverse trigonometric functions:

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

- Trigonometric Identities:

$$\sin^2 x + \cos^2 x = 1, \quad \tan^2 x + 1 = \sec^2 x, \quad \sin^2 x = \frac{1-\cos 2x}{2}, \quad \cos^2 x = \frac{1+\cos 2x}{2}, \quad \sin 2x = 2\sin x\cos x$$

- Partial fraction decompositions: let $P(x)/Q(x)$ be a rational function, where the degree of P is less than the degree of Q.
  - A single linear factor $x-a$ of $Q(x)$ contributes a term $\frac{A}{x-a}$ to the partial fraction decomposition of $P(x)/Q(x)$.
  - A repeated linear factor $(x-a)^n$ of $Q(x)$ contributes terms

$$\frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \ldots + \frac{A_n}{(x-a)^n}.$$

  - A single irreducible quadratic factor ax^2+bx+c of $Q(x)$ contributes a term $\frac{Ax+B}{ax^2+bx+c}$.
  - A repeated quadratic factor $(ax^2+bx+c)^n$ of $Q(x)$ contributes terms

$$\frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \ldots + \frac{A_nx+B_n}{(ax^2+bx+c)^n}.$$

- Parametric derivatives:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\big(dy/dx\big)}{dx/dt} \quad \text{whenever} \quad \frac{dx}{dt} \neq 0.$$

- Parametric arc length and surface area:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt \quad \text{and} \quad A = \int_a^b 2\pi y(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt.$$

Part I: Techniques of Integration. Please compute the following integrals.

1. $\int_{e}^{e^2} x \ln(x^4) dx$

2. $\int \frac{x+1}{2x^2+1} dx$

3. $\int_{0}^{\frac{\pi}{4}} \sin^2 \theta \cos^2 \theta d\theta$

4. $\int \frac{1}{(x+4)(x-1)} dx$

5. $\displaystyle\int \frac{7x^2 + x + 3}{(2x^2 + 1)(x + 1)} dx$

6. $\displaystyle\int \sin^{-1} x dx$

7. $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2} dx$ (Hint: break the integral into two pieces, one from $-\infty$ to 0, and the other from 0 to ∞.)

Part II: Parametric Equations.

8. Consider the parametric equations $x(t) = 1-2t$, $y(t) = t^3$, for $-1 \le t \le 1$. Sketch the parametric curve, including arrows to show motion along the curve as t increases. Solve for y in terms of x. Compute the derivatives $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

9. Compute the arc length of the parametric curve $x(t) = \sin t$, $y(t) = \dfrac{1}{2}\sin^2 t$ from $t = 0$ to $t = \dfrac{\pi}{2}$.

10. Compute the surface area of the surface of revolution given by rotating the parametric curve $x(t) = \dfrac{t^2}{2}$, $y(t) = t$ from $t = 0$ to $t = 1$ about the x-axis.

BONUS:

11. Show that $\int_{\frac{\pi^2}{16}}^{\frac{\pi^2}{4}} \frac{1}{\sqrt{x}} e^{-\tan\sqrt{x}} \left(1 + \tan^2\sqrt{x}\right) dx = \frac{2}{e}$