Differential Calculus and Characteristic Classes in Algebraic Geometry

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Calcul différentiel et classes caractéristiques en géométrie algébrique

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1 Introduction

The starting point of this work is an attempt to generalize the "Cauchy formula"

$$\mathrm{rg}_{\mathbb{C}}\mathbb{C}[z_1, ..., z_n]/(f_1, ..., f_n) = \frac{1}{(2i\pi)^n} \int_{|f_i|=\epsilon} \frac{df_1 \wedge ... \wedge df_n}{f_1 ... f_n}$$

where $f_1, ..., f_n$ are convergent series in n variables whose common vanishing locus is the origin.

If a collection of analytic functions has only the origin as its common vanishing locus, there must be *at least* n functions in the collection. The study of such collections involving more than n functions arises naturally (for example, in intersection theory), and it is reasonable to wonder if there still exists an explicit integral representation for

$$\mathrm{rg}_{\mathbb{C}}\mathbb{C}[z_1, ..., z_n]/(f_1, ..., f_s), \ \ s > n$$

which is *finite* (this condition is a version of the Nullstellensatz), provided that the variety of zeros of $f_1, ..., f_s$ is the origin.

In the algebraic context, such considerations lead naturally to the theory of Grothendieck duality. Recall that if A is a quotient of $\mathbb{C}[z_1, ..., z_n]$ which is also an artinian ring, then denoting by Ω^n the module of holomorphic differentials, there is a perfect pairing:

$$\theta : \mathrm{Ext}^n(A, \Omega^n) \times A \to \mathbb{C}$$

Identifying $\operatorname{Ext}^n(A, \Omega^n)$ with the dual $A^\vee$ (as finite-dimensional $\mathbb{C}$-vector spaces), the preceding formula may be written:

$$\theta\left\langle\left[\begin{smallmatrix} df_1\wedge\ldots\wedge df_n \\ f_1,\ldots,f_n \end{smallmatrix}\right], 1\right\rangle = \operatorname{rg}_{\mathbb{C}}\mathbb{C}[z_1,\ldots,z_n]/(f_1,\ldots,f_n)$$

where the *Grothendieck symbol* $\left[\begin{smallmatrix} df_1\wedge\ldots\wedge df_n \\ f_1,\ldots,f_n \end{smallmatrix}\right]$ represents the element of $\operatorname{Ext}^n(\mathbb{C}[z_1,\ldots,z_n]/(f_1,\ldots,f_n), \Omega^n)$ calculated by means of the Koszul resolution $K_\bullet(f_1,\ldots,f_n)$ of $\mathbb{C}[z_1,\ldots,z_n]/(f_1,\ldots,f_n)$, whose image is the class of the $\mathbb{C}[z_1,\ldots,z_n]$-linear function

$$\bigwedge^n \mathbb{C}[z_1,\ldots,z_n] \to \Omega^n$$
$$\epsilon_1\wedge\ldots\wedge\epsilon_n \mapsto df_1\wedge\ldots\wedge df_n$$

modulo $\operatorname{Hom}(\bigwedge^n \mathbb{C}[z_1,\ldots,z_n], \Omega^n)$.

In more general language, A has a resolution of length n by free $\mathbb{C}[z_1,\ldots,z_n]$-modules:

$$0 \to L_n \to L_{n-1} \to \ldots \to \mathbb{C}[z_1,\ldots,z_n] \to A \to 0$$

which determines a $\mathbb{C}[z_1,\ldots,z_n]$-linear function $L_n \to \Omega^n$ whose class modulo $\operatorname{Hom}(L_n, \Omega^n)$, denoted by ω, is such that

$$\theta\langle\omega, 1\rangle = \operatorname{rg}_{\mathbb{C}} A$$

After some experimentation (for small codimension, using the connecting theorems of Gaeta, Peskine, Szprio, Buschbaum, and Eisenbud), we find in fact that from a free resolution of A, we may obtain, by means of a very simple procedure involving differentials, a canonical element of $\operatorname{Ext}^n(A, \Omega^n)$ (independent of the choice of resolution), which solves the problem. We also observe that the same procedure permits us, quite generally, to associate to any module M of finite type over a noetherian k-algebra R (not necessarily smooth), corresponding classes in $\operatorname{Ext}^i(M, \Omega^i_{R/k}\otimes_R M)$ for $i \geq 1$.

The method is the following:

Let

$$\ldots \longrightarrow L_i \xrightarrow{\phi_i} L_{i-1} \longrightarrow \ldots \longrightarrow L_1 \xrightarrow{\phi_1} L_0 \longrightarrow M \longrightarrow 0$$

be a resolution of M by free R-modules. Choose bases (e), (f) for the free modules L_1 and L_0 such that

$$\phi_1(e_j) = \sum_i a_{ij} f_i$$

Differentiating the coefficients of the matrix A_1 representing ϕ_1 with respect to these bases, we obtain an R-linear function $L_1 \to \Omega^1_{R/k} \otimes L_0$ defined by the formula

$$d\phi_1(e_j) = \sum_i da_{ij} f_i$$

Of course, the function defined by this procedure depends on the choice of bases for L_1 and L_0. More precisely, if $A_1' = P \cdot A_1 \cdot Q^{-1}$ is the matrix representing ϕ_1 with respect to a different choice of bases (e') for L_1 and (f') for L_0, then

$$dA_1' = P \cdot \left(P^{-1} \cdot dP \cdot A_1 + dA_1 + A_1 \cdot dQ^{-1} \cdot Q\right) \cdot Q^{-1}$$

Thus, (expressing everything in terms of the original bases (e) and (f)), we see that the change of basis alters the matrix representation of the function $L_1 \to \Omega^1_{R/k} \otimes L_0$ by the addition of two matrices, $A_1 \cdot dQ^{-1} \cdot Q$ and $P^{-1} \cdot dP \cdot A_1$. The first of these extra terms, $A_1 \cdot dQ^{-1} \cdot Q$, may be eliminated by considering instead the *composition*

$$L_1 \to \Omega^1_{R/k} \otimes L_0 \to \Omega^1_{R/k} \otimes M$$

induced by the map $L_0 \to M$, since $A_1 \cdot dQ^{-1} \cdot Q$ factors through the map $\Omega^1_{R/k} \otimes L_1 \to \Omega^1_{R/k} \otimes L_0$. The second extra term, $P^{-1} \cdot dP \cdot A_1$, may be eliminated by considering the function *modulo* $\mathrm{Hom}(L_0, \Omega^1_{R/k} \otimes M)$, since $P^{-1} \cdot dP \cdot A_1$ factors through ϕ_1. Finally, if $x = \sum_i x_i e_i \in \mathrm{Ker}\phi_1$, then by differentiating the identity $A_1[x] = 0$ (where $[x]$ is the column vector corresponding to x in the (e) basis), we see that the composition $L_1 \to \Omega^1_{R/k} \otimes L_0 \to \Omega^1_{R/k} \otimes M$ sends x to zero. Thus, the element of $\mathrm{Ext}^1(M, \Omega^1_{R/k} \otimes M)$ defined by this procedure is independent of the choice of bases.

Similarly, if we now consider the first i steps of the resolution of M, choose bases for the modules $L_0, ..., L_i$, and differentiate the coefficients of the matrices $A_1, ..., A_i$ representing the homomorphisms $\phi_1, ..., \phi_i$ with respect to the chosen bases, we obtain, by considering the function

$$L_i/\mathrm{Ker}\phi_i \xrightarrow{dA_1 \cdot \ldots \cdot dA_i} \left(\Omega^1_{R/k}\right)^{\otimes i} \otimes L_0 \longrightarrow \left(\Omega^1_{R/k}\right)^{\otimes i} \otimes M$$

modulo $\mathrm{Hom}\left(L_{i-1}, \left(\Omega^1_{R/k}\right)^{\otimes i} \otimes M\right)$, an element $c^i \in \mathrm{Ext}^i(M, \left(\Omega^1_{R/k}\right)^{\otimes i} \otimes M)$; i.e. the element c^i is independent of both the choice resolution of M and the choice of bases for a particular resolution.

Finally, we deduce the existence of classes $\gamma^i_M \in \mathrm{Ext}^i(M, \Omega^i_{R/k} \otimes M)$ in either of the two following situations:

1. Every integer is invertible in k. This condition arises because in this case γ^i_M is the image in $\mathrm{Ext}^i(M, \Omega^i_{R/k} \otimes M)$ of $\frac{c^i}{i!}$. The necessity of dividing by the factorial will be discussed in due course.

2. R is *differentiably smooth* over k.

We show that under these circumstances, c^i corresponds to a canonical element of $\operatorname{Ext}^i(M, A_i(\Omega^1_{R/k}) \otimes M)$, where $A_i(\Omega^1_{R/k}) \hookrightarrow (\Omega^1_{R/k})^{\otimes i}$ is the antisymmetric tensors. Identifying $A_i(\Omega^1_{R/k})$ with $\Omega^i_{R/k}$, we obtain the desired class $\gamma^i_M \in \operatorname{Ext}^i(M, \Omega^i_{R/k} \otimes M)$.

We also show that if M has a resolution by free R modules of length r, then $\operatorname{Ext}^r(M, \Omega^r_{R/k} \otimes M) \cong \operatorname{Ext}^r(M, \Omega^r_{R/k}) \otimes M$, which assures that in the example considered previously, in which A is artinian, γ^n_A may be considered as an element of $\operatorname{Ext}^n(A, \Omega^n_{A/k}) \otimes A = \operatorname{Ext}^n(A, \Omega^n_{A/k})$. This is the element we were originally seeking. The reader may verify that if $R = \mathbb{C}[z_1, ..., z_n]$ and $I = (f_1, ..., f_r)$, where $f_1, ..., f_r$ is a regular sequence, then

$$c^r_{R/I} = \left[\begin{array}{c} \sum_{\sigma \in S_r} \epsilon_\sigma df_{\sigma(1)} \wedge ... \wedge df_{\sigma(r)} \\ f_1, ..., f_r \end{array} \right]$$

and

$$\gamma^r_{R/I} = \left[\begin{array}{c} df_1 \wedge ... \wedge df_r \\ f_1, ..., f_r \end{array} \right].$$

In particular, if $I = (f_1, ..., f_n)$, then

$$\gamma^r_{R/I} = \left[\begin{array}{c} df_1 \wedge ... \wedge df_n \\ f_1, ..., f_n \end{array} \right].$$

We now look at an example showing how, beginning with the class γ^n_A, we can determine the length of the artinian ring A by means of "integration." Let

$$R := \mathbb{C}[z_1, z_2], \; I := (z_1^2, z_1 z_2, z_2^2), \; A := R/I$$

and consider the resolution of A by free R-modules given by

$$0 \longrightarrow R^2 \xrightarrow{\phi_2} R^3 \xrightarrow{\phi_1} R \longrightarrow A \longrightarrow 0$$

where ϕ_2 and ϕ_1 are represented by the matrices

$$\begin{bmatrix} z_2 & 0 \\ -z_1 & z_2 \\ 0 & -z_1 \end{bmatrix}$$

and

$$\begin{bmatrix} z_1^2 & z_1 z_2 & z_2^2 \end{bmatrix}$$

in the obvious bases (in which the basis vectors are the elements 1 in each factor of R). We call these bases (f_1, f_2) and (e_1, e_2, e_3); in this notation,

$$f_1 \text{ corresponds to } \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

for example. Then γ_A^2, as an element of $\mathrm{Hom}(R^2, \Omega^2_{R/k})/\mathrm{Hom}(R^3, \Omega^2_{R/k})$, is the class of the function sending f_1 to $\eta_1 := -\frac{3}{2} z_1 dz_1 \wedge dz_2$ and f_2 to $\eta_2 := -\frac{3}{2} z_2 dz_1 \wedge dz_2$.

Let B be the complete intersection

$$B := \mathbb{C}[z_1, z_2]/(z_1^2, z_2^2)$$

There is a commutative diagram

$$\begin{array}{ccccccc}
R^2 & \xrightarrow{\phi_2} & R^3 & \xrightarrow{\phi_1} & R & \longrightarrow & A \\
\uparrow{\scriptstyle \theta_2} & & \uparrow{\scriptstyle \theta_1} & & \uparrow{\scriptstyle Id} & & \uparrow \\
R & \xrightarrow[\phi_2']{} & R^2 & \xrightarrow[\phi_1']{} & R & \longrightarrow & B
\end{array}$$

where

$$\begin{cases}
\phi_1'(\epsilon_1) = z_1^2, \ \phi_1'(\epsilon_2) = z_2^2; \\
\phi_2'(\epsilon_1 \wedge \epsilon_2) = z_1^2 \epsilon_2 - z_2^2 \epsilon_1 \\
\theta_1(\epsilon_1) = e_1, \ \theta_1(\epsilon_2) = e_3 \\
\theta_2'(\epsilon_1 \wedge \epsilon_2) = z_1 f_2 - z_2 f_1
\end{cases}$$

(Here, ϵ_1 and ϵ_2 represent the units in the two factors of R appearing in the term R^2 in the middle of the bottom row of the commutative diagram, and the copy of R mapping into R^2 from the left is thought of as the wedge product of these two copies of R, with basis element $\epsilon_1 \wedge \epsilon_2$.)

Let $[\alpha_1, \alpha_2]$ be the element of $\mathrm{Ext}^2(A, \Omega^2_{R/k})$ which is the image of the function sending f_i to α_i (PROBLEM, α_i undefined). $\mathrm{Ext}^2(A, \Omega^2_{R/k})$ may be identified as a sub-module of $\mathrm{Ext}^2(B, \Omega^2_{R/k})$ by the function sending $[\alpha_1, \alpha_2]$ to $\begin{bmatrix} -z_2\alpha_1 - z_1\alpha_2 \\ z_1, z_2 \end{bmatrix}$. Thus

$$\mathrm{Res}\begin{bmatrix} -z_2\eta_1 - z_1\eta_2 \\ z_1, z_2 \end{bmatrix} = \mathrm{Res}\begin{bmatrix} 3z_1 z_2 dz_1 \wedge dz_2 \\ z_1, z_2 \end{bmatrix} = 3.$$

At this point, we have at our disposal a *local theory of fundamental classes* associated with a module M over a k-algebra, along with an interpretation of the class γ_M^n in the case where M is the [local] ring of a closed point of a smooth variety of dimension n.

The identification of γ^r_A, where A is the [coordinate or local?] ring of a Cohen-Macaulay subspace X of codimension r in a variety Z of dimension n, reduces essentially to a continuous family of data like that appearing in the "Cauchy formula." (?) We project X onto a smooth subspace W of the same dimension as X in Z, by means of a finite, flat morphism ([we call this] continuity) (?), which is possible if and only if X is Cohen-Macaulay; in other words, we view A as a finite, flat B-module. The natural generalization [of what, exactly?] is the commutativity of the following diagram:

$$\begin{array}{ccccc} A \otimes_B \Omega^{n-r}_{B/k} & \longrightarrow & A \otimes_R \Omega^{n-r}_{R/k} & \longrightarrow & \mathrm{Ext}^r(A, \Omega^n_{R/k}) \\ \big\downarrow Tr \otimes Id & & & & \big\downarrow Tr \\ \Omega^{n-r}_{B/k} & & \longrightarrow & & \Omega^{n-r}_{B/k} \end{array}$$

where the map $A \otimes_R \Omega^{n-r}_{R/k} \to \mathrm{Ext}^r(A, \Omega^n_{R/k})$

is uniquely determined by γ^r_A. For obvious reasons the classes $\gamma^r_{\mathcal{O}_{X,x}}$ [together define?] an element of $\mathrm{Ext}^r(\mathcal{O}_X, \Omega^r_{Z/k}) \hookrightarrow H^r_{|Z|}(\Omega^r_{Z/k})$, the fundamental class of X. The *relative* character of of the definition of these classes allows us to obtain a general existence theorem for the relative fundamental classe of a flat family $X \to S$. (We go from the flat case with Cohen-Macaulay fibers to the flat case in general and show that the sheaf $\mathcal{E}^r(\mathcal{O}_X, \Omega^r_{Z/k})$ has sufficient depth). (?)

Another generalization of the "Cauchy formula" also arises naturally in the calculation of of the Euler characteristic of a complex with cohomology vanishing above a given finite degree.

The construction of classes [in the Euler characteristic setting just mentioned?] is not much different than the previous theory for complexes of modules bounded above (for notational purposes, we assume $M_n = 0$ for $n > 0$). In this case, we obtain classes $\gamma^i_{M^\bullet}$ in
$\mathrm{Ext}^i(M^\bullet, \Omega^i_{R/k} \otimes^L_R M^\bullet)$, where $\otimes^L_R$ is the derived tensor product. (We must take care [because $\Omega^1_{R/k}$ is not a flat R-module](?), and if $M^\bullet$ is a complex concentrated in degree zero, the present construction does not coincide exactly with the one above.)

We show that if R is a smooth local ring of dimension n over a field k with residue field finite over k, we can define an *evaluation:*

$$e : \mathrm{Ext}^i(M^\bullet, \Omega^n_{R/k} \otimes^L_R M^\bullet) \to k$$

such that

$$e(\gamma^n_{M^\bullet} = \sum_i (-1)^i \mathrm{rg} H^i(M^\bullet).$$

This formula is a local form of the Riemann-Roch theorem. This result permits us to calculate the *intersection number* of two R-modules M and N [provided length$(M \otimes_R N)$ is finite](?) by means of the fundamental classes γ_M^i and γ_N^j of the two modules and the above evaluation. More precisely, $e(M,N)$ (defined by Serre as being the Euler characteristic of $M \otimes^L N$) is given by the formula:

$$e(M,N) = \sum_{i+j=n} e(\gamma_M^i \cdot \gamma_N^j)$$

where $\gamma_M^i \cdot \gamma_N^j \in \mathrm{Ext}^n(M \otimes N, \Omega^n_{R/k} \otimes M \otimes N)$ is the *cup product* of γ_M^i and γ_N^j.

This then leads to the conclusion that if X_1 and X_2 are two subspaces of complementary dimensions r_1 and r_2 in a smooth variety Z of dimension n (i.e $n = r_1 + r_2$) intersecting properly in a point x, then the local intersection number $(X_1 \cdot X_2)_x$ of X_1 and X_2 is given by the formula

$$(X_1 \cdot X_2)_x = e(C_{X_1} \cup C_{X_2})$$

where $C_{X_i} \in H^{r_i}_{|X_i|}(\Omega^n_{Z/k})$ is the fundamental class of X_i for $i = 1, 2$, $C_{X_1} \cup C_{X_2}$ is the cup product of the two classes in $H^n_x(\Omega^n_{Z/k})$, and $e : H^n_x(\Omega^n_{Z/k}) \to k$ is the evaluation or *residue morphism* sending the local cohomology to the base field k.

In the classical situation where X_1 and X_2 are local complete intersections of complementary dimensions intersecting at a point $x \in Z$, where $f_1, ..., f_{r_1}$ and $g_1, ..., g_{r_1}$ are the regular sequences defining X_1 and X_2 in a neighborhood of x, the local intersection number is given by the following integral:

$$(X_1 \cdot X_2)_x = \frac{1}{2i\pi)^n} \int_{|f_i|=|g_j|=\epsilon} \frac{df_1 \wedge ... \wedge df_{r_1} \wedge dg_1 \wedge ... \wedge dg_{r_2}}{f_1 ... f_{r_1} g_1 ... g_{r_2}}$$

Finally, these constructions globalize. We limit ourselves here to the algebraic case. The same procedure involving differentials, with free local resolutions now replaced by simplicial free resolutions or by locally free resolutions if they exist (cf. II.1.5), produce *global fundamental classes* $\lambda^i_{\mathcal{M}^\bullet}$ for $i \geq 1$, for every bounded-above complex $\mathcal{M}^\bullet$ of sheaves of $\mathcal{O}_X$-modules with coherent cohomology (again we assume vanishing in positive degrees; i.e. $\mathcal{M}^n = 0$ for all $n > 0$). (Note here that even if X is smooth over S, we cannot [as in the local case (?)] canoncially divide by $i!$ in positive characteristic.). Paradoxically, these global classes were identified previously (Atiyah, Illusie); so the second chapter of this book does not contain anything essentially new. However, the similarity of the constructions in the local and global cases, and the unified viewpoint thus afforded, is useful to stress and may prove fruitful for later work.

We illustrate this briefly in Appendix 2, which is devoted to a parallel examination of intersection problems in the local and global settings. The trace functions

$$Tr : \mathrm{Ext}^i(\mathcal{M}^\bullet, \Omega^i_{R/k} \otimes^L_R \mathcal{M}^\bullet) \to H^i(\Omega^i_{X/S})$$

defined by Illusie if $\mathcal{M}^\bullet$ is a perfect complex, as well as the traces *with support*

$$Tr^Z : \mathrm{Ext}^i(\mathcal{M}^\bullet, \Omega^i_{R/k} \otimes^L_R \mathcal{M}^\bullet) \to H^i_Z(\Omega^i_{X/S})$$

if $\mathcal{M}^\bullet$ is also acyclic on the complement of a closed subset Z, which send the fundamental classes $\lambda^i_{\mathcal{M}^\bullet}$ to the *Newton classes* $\nu^i_{\mathcal{M}^\bullet}$ and *Newton classes with support* $\nu^{Z,i}_{\mathcal{M}^\bullet}$, respectively, appear to serve as a substitute for a "moving lemma" for the intersection theory of singular varieties, with the fundamental classes generating the Newton classes while preserving information about the supports.

In particular, it is these formulas that relate the symmetric functions of Newton to the elementary symmetric functions that send the Newton classes to the Chern classes of a perfect complex. (?) Exploiting then the link between fundamental classes and the fundamental class of a subscheme Z of codimension p obtained in Chapter 1, we obtain a simple and new demonstration of the Grothendieck formula relating the fundamental class of a subscheme to the pth Chern class of the structure sheaf:

$$c^p(\mathcal{O}_Z) = (-1)^{p-1}(p-1)!\gamma^p_Z$$

The functorial behavior of Newton classes, Chern classes, and Chern characters follows from results of Illusie about the functorial behavior of trace maps. The analysis of the trace map

$$\mathrm{Hom}(\mathcal{K}^\bullet_1 \otimes^L \mathcal{E}^\bullet, \mathcal{K}^\bullet_1 \otimes^L \mathcal{E}^\bullet) \to \mathrm{Hom}(\mathcal{K}^\bullet_1, \mathcal{K}^\bullet_1)$$

under the general hypotheses used for simplicial free resolutions of $\mathcal{K}^\bullet_1$, $\mathcal{K}^\bullet_2$, and $\mathcal{E}^\bullet$ presents some difficulties because the necessary morphisms do not exist in general in the category of simplicial systems. [It is therefore particularly important in this case] (?) to present a complete exposition. However, in the case where the perfect complexes under consideration are quasi-isomorphic to bounded complexes of locally free sheaves of $\mathcal{O}_X$-modules, the trace maps are easy to analyze; the functorial properties [are straightforward] and consequently so is the identity between Chern classes obtained via fundamental classes and constructrions by Grothendieck for "vector bundles" on any nonsingular algebraic variety in characteristic zero. (cf [Ati] and [Groth]).

This book is divided into two Chapters. Chapter I is devoted to local theory, and Chapter II to global theory. (In Chapter II, references to Chapter I are numbered I...).

Summary of Chapter I:

- The first two sections are devoted to explaining the notions of of differentials of a morphism and compositions of differentials. These sections are mainly for the purpose of introducing the techniques of the calculus of differentials developed more systematically in succeeding sections[1].

[1] The projective case is developed in full, but the reader may choose to focus on the case of free modules.

- In Section 3, we define the fundamental classes $\gamma^i_{M^\bullet}$ for a complex $M^\bullet$ of R-modules of finite type, where R is a k-algebra.
- In Section 4, we consider the case where $\gamma^i_{M^\bullet}$ is a single module (i.e. a complex concentrated in degree zero) and where R is differentiably smooth over k.
- In section 5, we derive the *relative* fundamental classes of a locally noetherian scheme of finite type (respectively, an analytic space) which is flat over the base, by means of the classes already defined.
- In Section 6, the link with traces of differentials in the reference [Ang] is clarified.
- In Section 7, the functoriality of the fundamental classes $\gamma^i_{M^\bullet}$ is studied, (with respect to tensor product, inverse image, and exact sequences).
- In Section 8, we show how the evaluation (or image of the residue map) of the fundamental class of maximum rank for a complex $M^\bullet$ with bounded cohomology gives the Euler-Poincare characteristic. An appendix is devoted to [this relationship](?)

Summary of Chapter II:

- In Section 1, we construct the global fundamental classes of Atiyah.
- In Section 2, we construct the Newton classes and Chern classes for a perfect complex.
- In Section 3, some sample calculations of Chern classes and fundamental classes of subschemes are given.
- Section 4 is devoted to functorial properties of these classes. We use the characterization of the fundamental class of a subscheme from Chapter I to show directly some classical results for Chern classes of vector bundles; for example, the formula for self-intersection.
- In a short appendix, we sketch some consequences for local and global intersection theory.