Graphics for *Foundations of Causal Theory I*

Benjamin F. Dribus

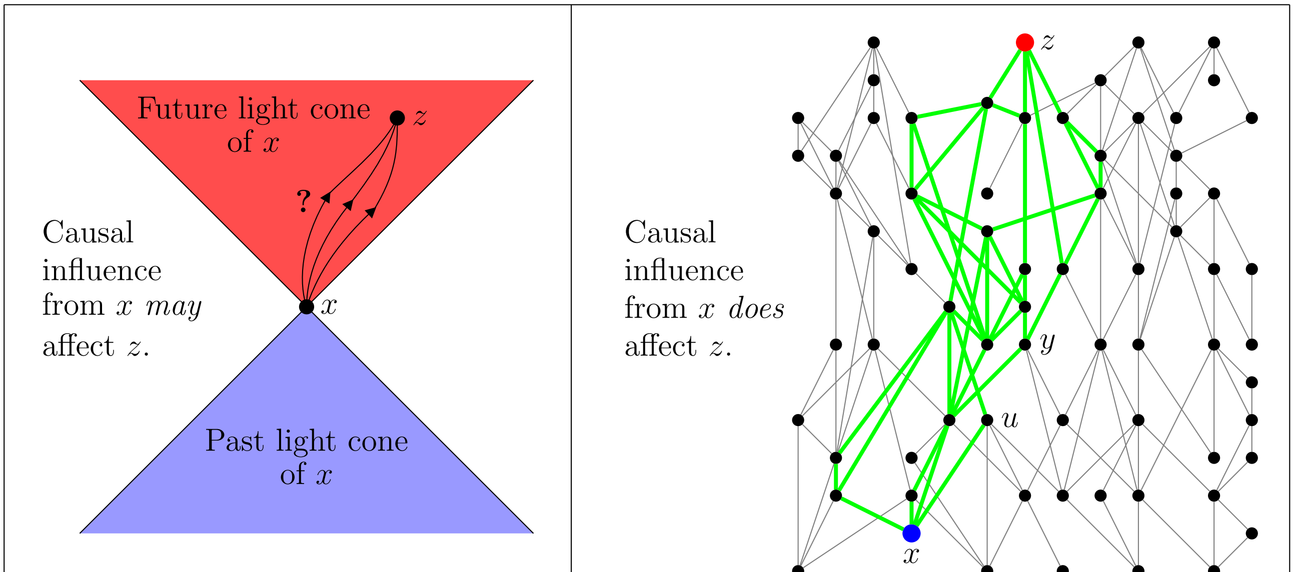


FIGURE 1

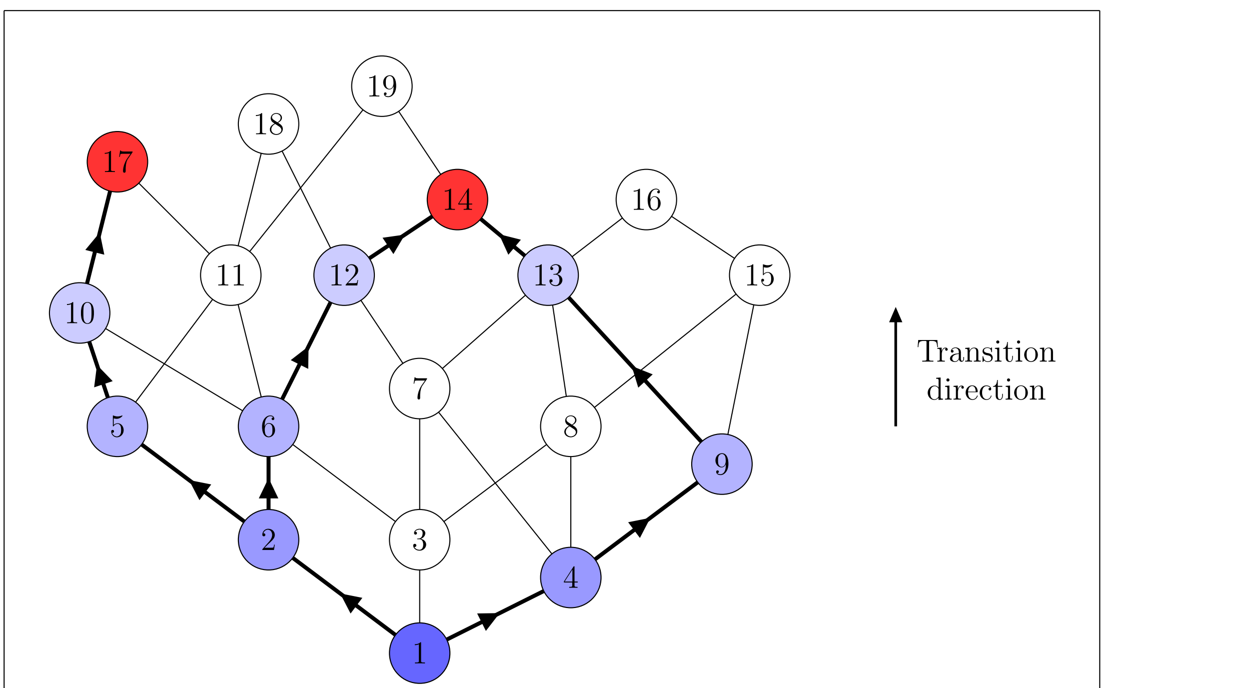
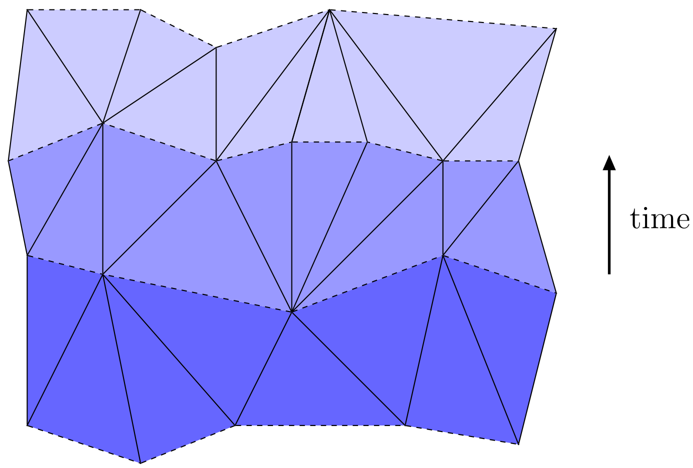
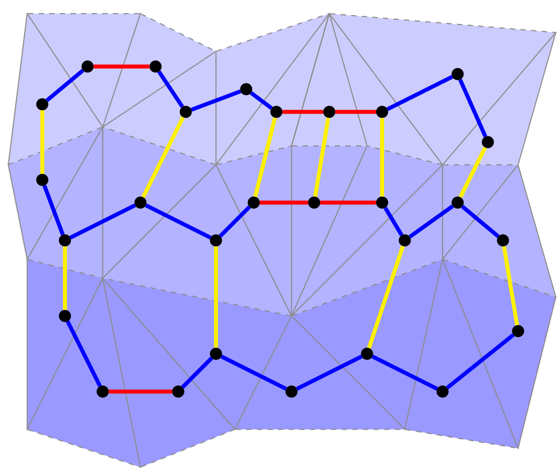


FIGURE 4



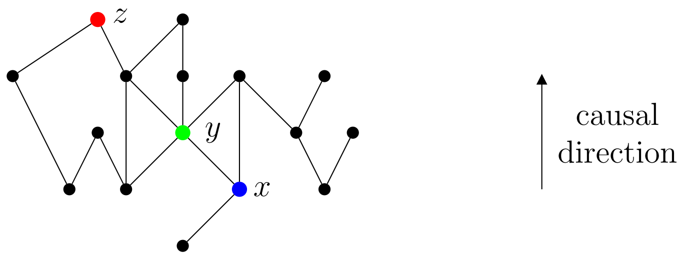
2-D causal dynamical triangulation

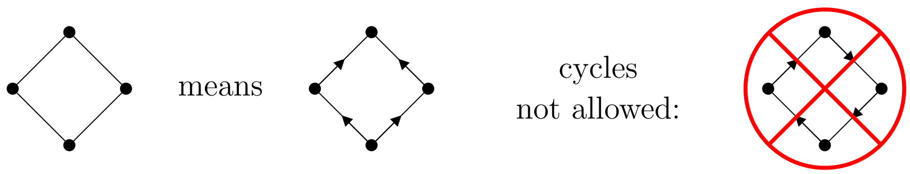
BDDCT

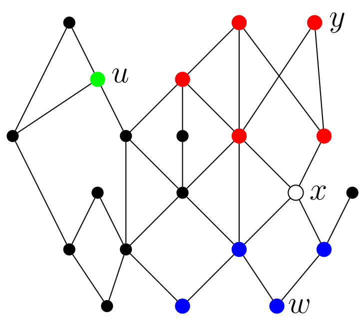


Colored Poincare dual graph for a 2-D causal dynamical triangulation.

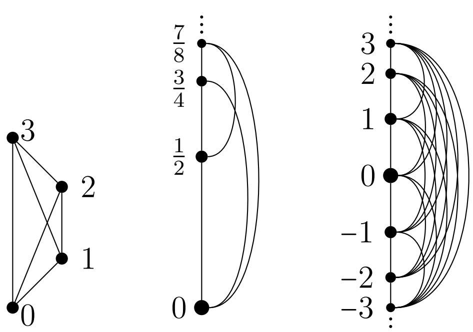
BDDCT



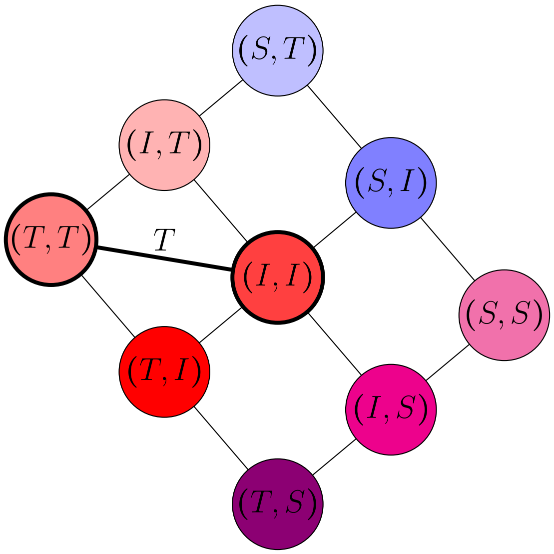


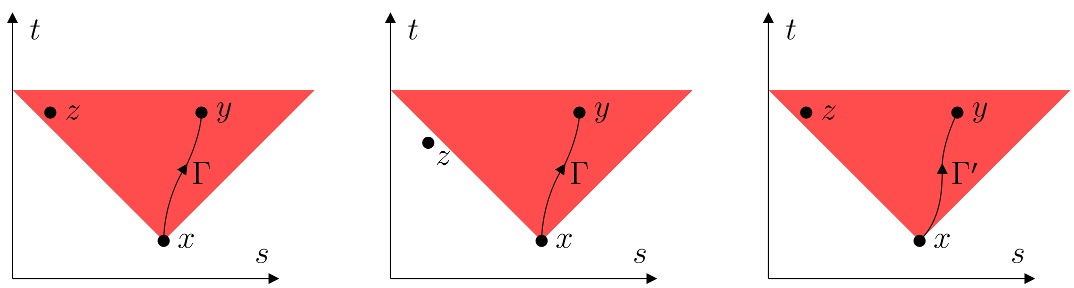


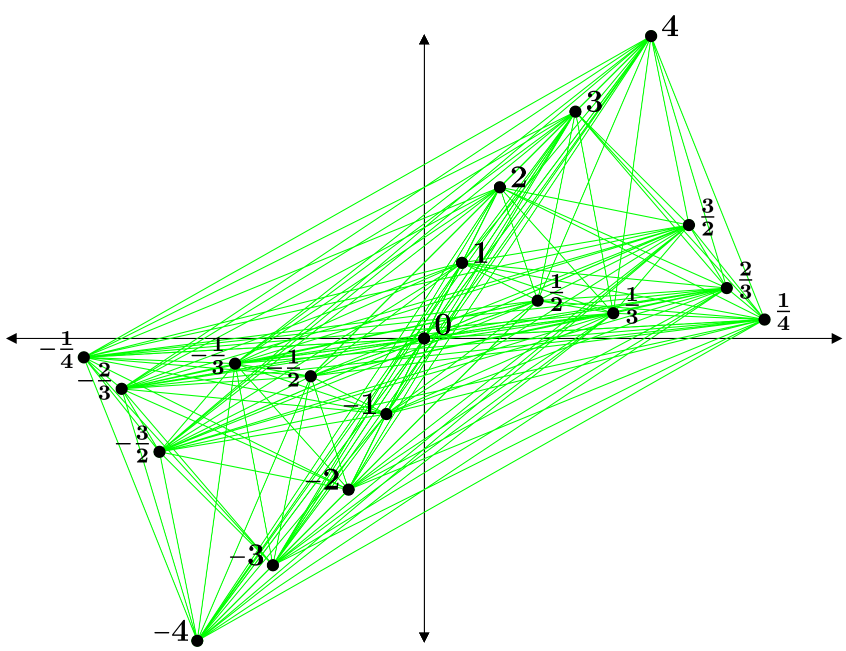
In this diagram, x is considered to be in the "present." Note how the elements in the past of x are colored blue, and the elements in the future of x are colored red. The distinguished element u, which is unrelated to x, is colored green. Also note how the distinguished elements are indicated by slightly larger nodes.

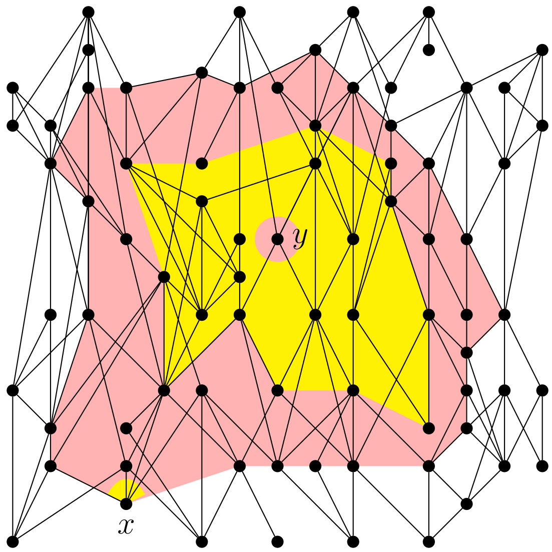


These diagrams represent the set $\{0, 1, 2, 3\}$, the set $\{\frac{2^n-1}{2^n} | n = 0, 1, 2, ...\}$, and the integers, all of which are transets with the usual order inherited from the rational numbers. In these diagrams, the labels on the elements are canonical, because the elements themselves are numbers. This is usually not the case.

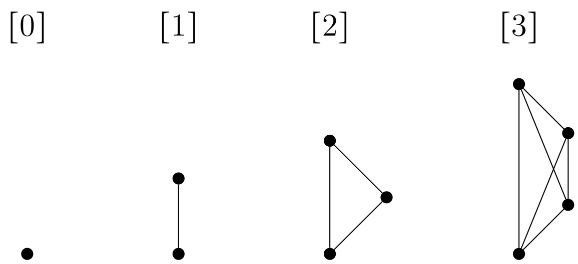

(S,T)
(I,T)
(S,I)
(T,T)
T
(I,I)
(S,S)
(T,I)
(I,S)
(T,S)


t
z
y
Γ
x
s
t
y
z
Γ
x
s
t
z
y
Γ'
x
s

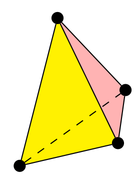

4
3
2
$\frac{3}{2}$
1
$\frac{1}{2}$
$\frac{2}{3}$
$\frac{1}{3}$
$\frac{1}{4}$
0
$-\frac{1}{4}$
$-\frac{1}{3}$
$-\frac{1}{2}$
$-\frac{2}{3}$
−1
$-\frac{3}{2}$
−2
−3
−4

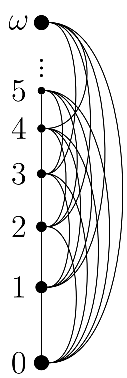


Note that the element x is in the interior of the subset even though it “looks like it is near the edge of the subset,” and the element y is in the boundary even though it “looks like it is in the interior of the subset.”

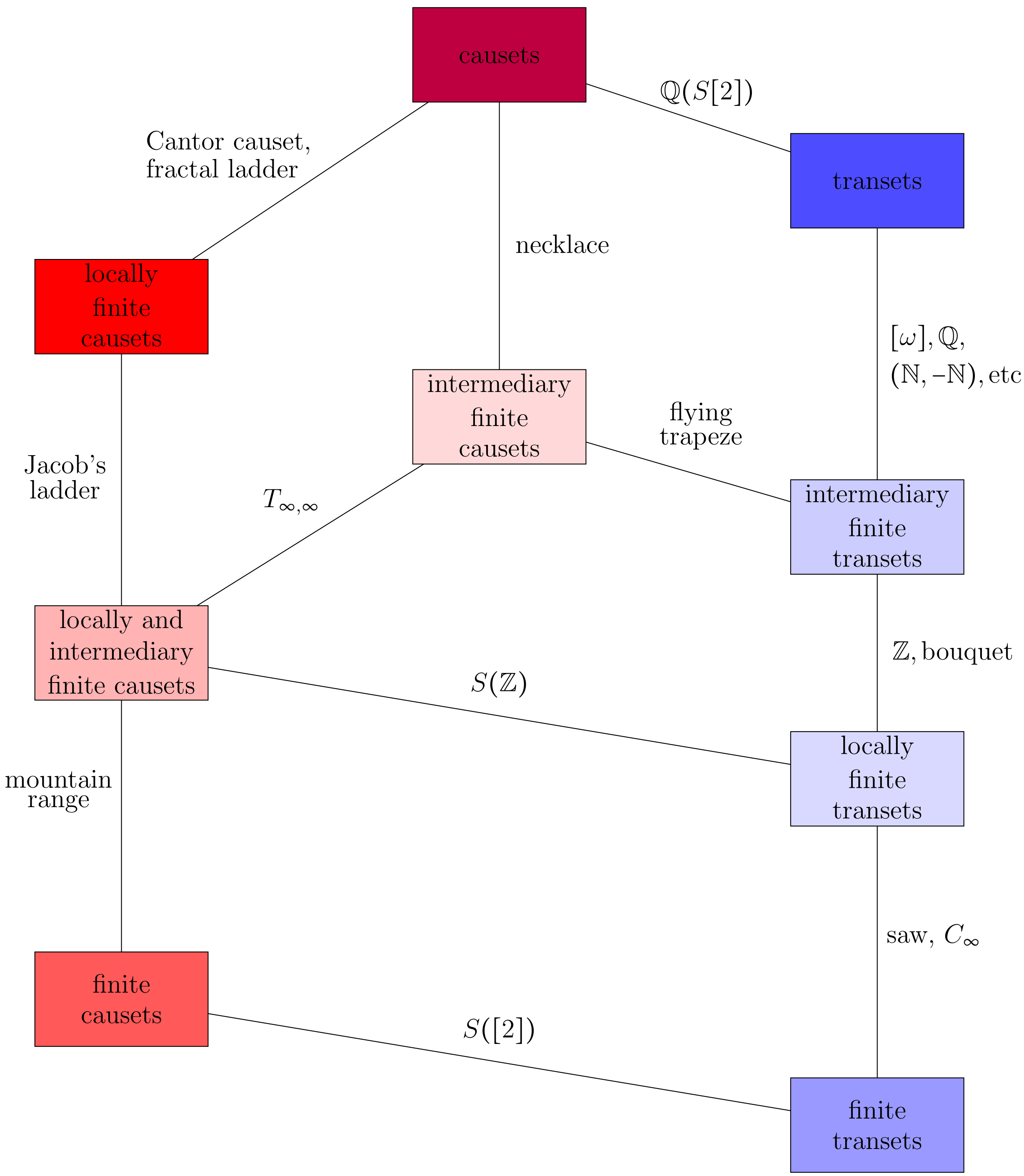


[3] as a geometric simplex



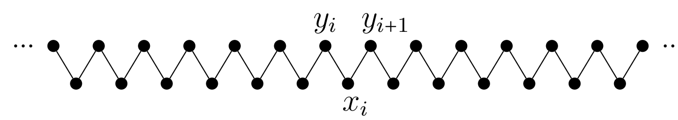


Cantor’s first transfinite ordinal
as a causet $[\omega]$


causets
$\mathbb{Q}(S[2])$
transets
Cantor causet,
fractal ladder
necklace
locally
finite
causets
$[\omega], \mathbb{Q},$
$(\mathbb{N}, -\mathbb{N})$, etc.
intermediary
finite
causets
flying
trapeze
Jacob's
ladder
$T_{\infty,\infty}$
intermediary
finite
transets
locally and
intermediary
finite causets
$\mathbb{Z}$, bouquet
$S(\mathbb{Z})$
locally
finite
transets
mountain
range
saw, C_∞
finite
causets
$S([2])$
finite
transets

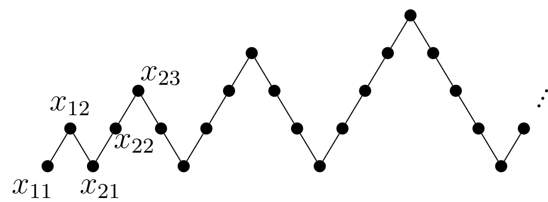
$S([2])$ is the skeleton of the 2-simplex. It is the simplest intransitive causet.

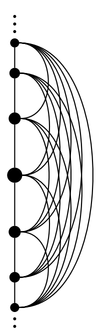
The **saw** is the causet with elements x_i, y_j, $i, j \in \mathbb{Z}$, and relations $x_i < y_i, y_{i+1}$. Any causet with chains of length at most 1 is transitive.



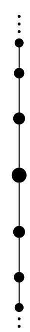
C_∞ is the causet with a countable number of elements and no relations. Similarly, C_n is the causet with n elements and no relations.

The **mountain range** is the causet with elements x_{ij}, $i, j = 1, 2, 3, \ldots$, $1 \le j \le 2i$ and relations $x_{ij} < x_{i,j+1}$, for $1 \le j < i$, $x_{i,j+1} < x_{ij}$ for $i \le j < 2i$, and $x_{i1} < x_{i-1,2i-2}$, for $i > 1$.



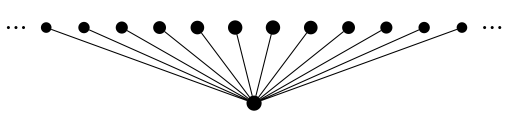


The **integers** $\mathbb{Z}$ as a causet.

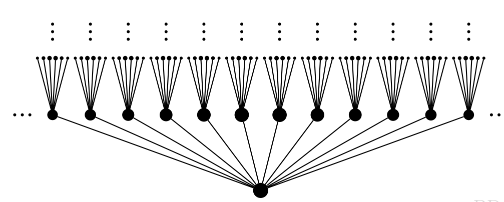


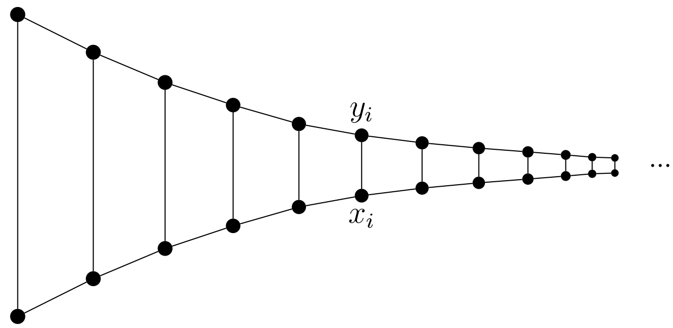
The skeleton $S(\mathbb{Z})$ of the integers.

The **bouquet** was defined in a previous subsection.



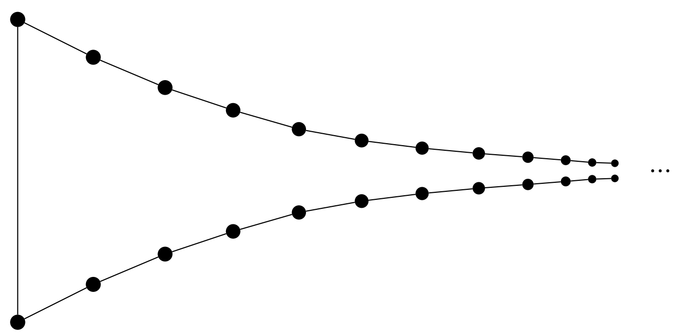
$T_{\infty,\infty}$ is the tree with an infinite number of "levels" and an infinite number of branches on each element at each level.





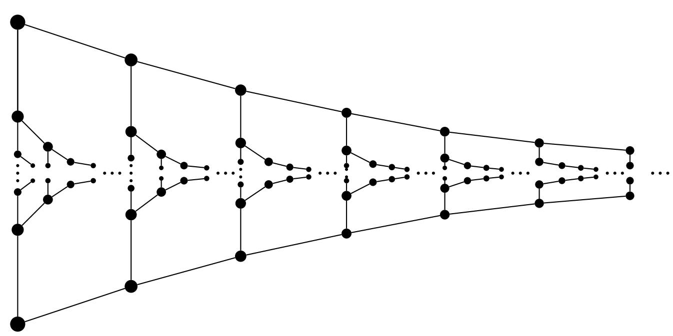
Jacob's ladder was defined in a previous subsection.

BDCST



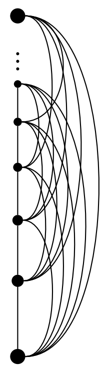
The **flying trapeze** is the causet obtained by removing the relations $x_i < y_i$, $i > 1$ from Jacob's ladder.

BDCST

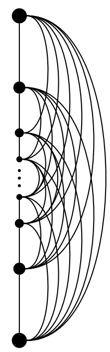


The **fractal ladder** is the causet obtained by attaching a new copy of Jacob's ladder to each "rung" of Jacob's ladder, and so on ad infinitum.

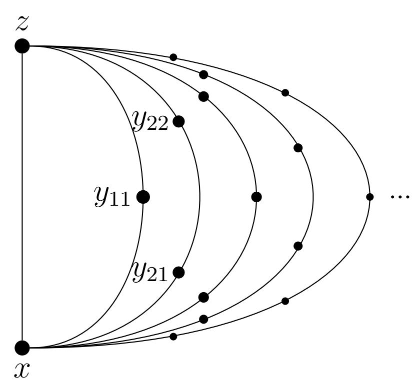
BDCST



The transet $[\omega]$ corresponding to Cantor's first transfinite ordinal was defined in a previous section.



The transet $(\mathbb{N}, -\mathbb{N})$ is defined by taking disjoint copies of $\mathbb{N}$ and $-\mathbb{N}$ and defining new relations so that every element of $\mathbb{N}$ precedes every element of $-\mathbb{N}$. This construction applies more generally; for example, $[\omega] = (\mathbb{N}, [0])$, and $[5] = ([2], [1], [0]) = ([1], [1], [1])$, etc.

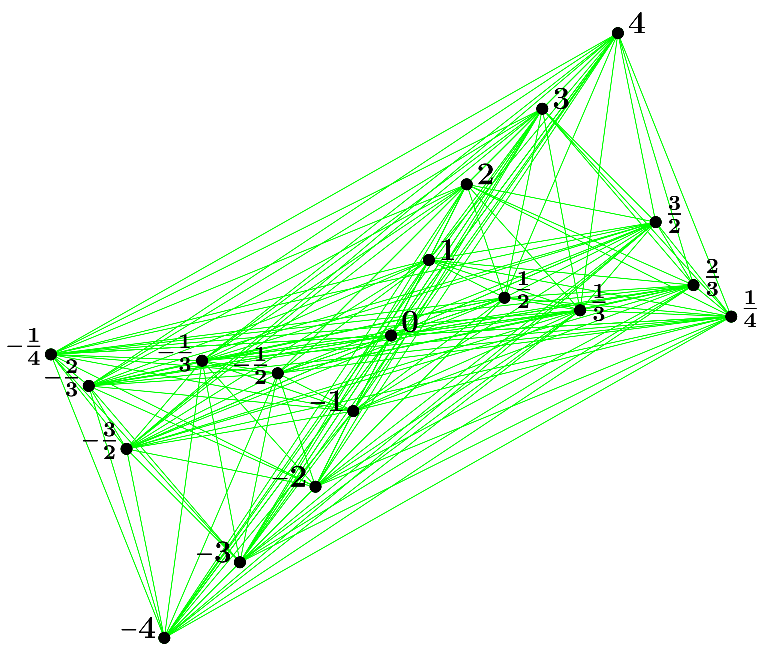


The **necklace** is the causet with elements x, z, and y_{ij} for $0 < j < i$ and relations

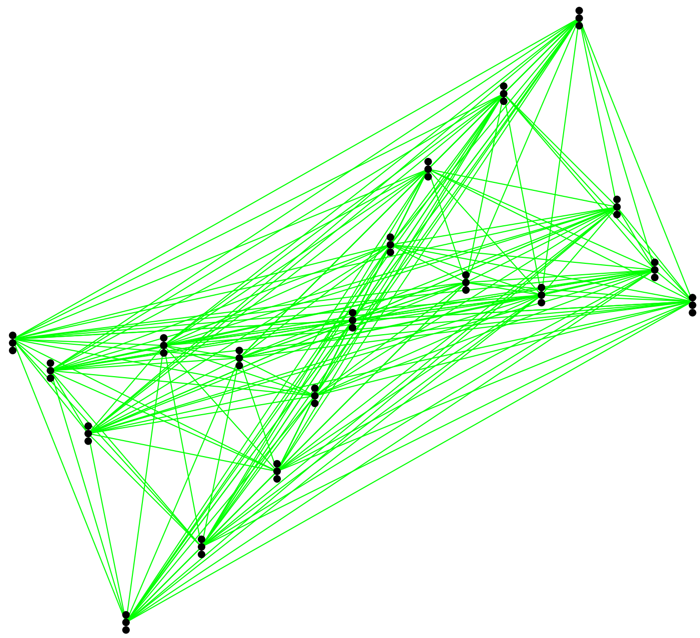
$x < z$, $x < y_{i1}$ for all i

and

$y_{ij} < y_{i,j+1}$ for $j < i$, and $y_{ii} < z$.

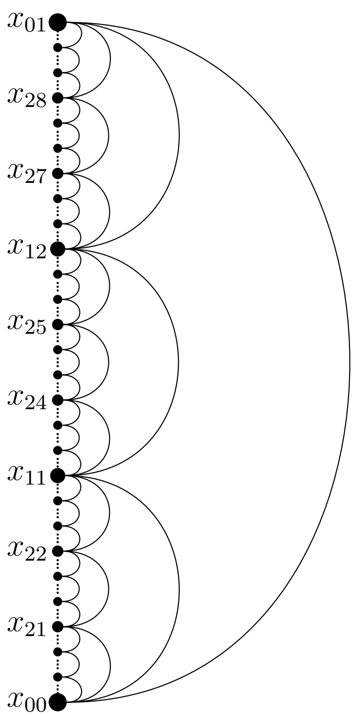


This is the partial Hasse diagram of $\mathbb{Q}$ from section 1.2



$\mathbb{Q}(S([2]))$ is the causet obtained by replacing every element of $\mathbb{Q}$ with the causet $S([2])$, and defining relations $a_p < b_q$ between each pair a_p, b_q of elements of the copies of $S([2])$ corresponding to rationals $p < q$. This construction applies more generally; for example, $[3] = [1]([1])$ and $\mathbb{N} = \mathbb{N}([2])$.

BDCST



The **Cantor causet** is the causet with elements

$$\{x_{ij} | i, j \in \mathbb{N}, 0 \le j \le 3^i, j \ne 0 \text{ modulo } i \text{ for } i > 0\}$$

and relations

$$x_{ij} < x_{i+k, 3^k j+1} \text{ for all } k \ge 0 \text{ if } j = 1 \text{ modulo } 3$$

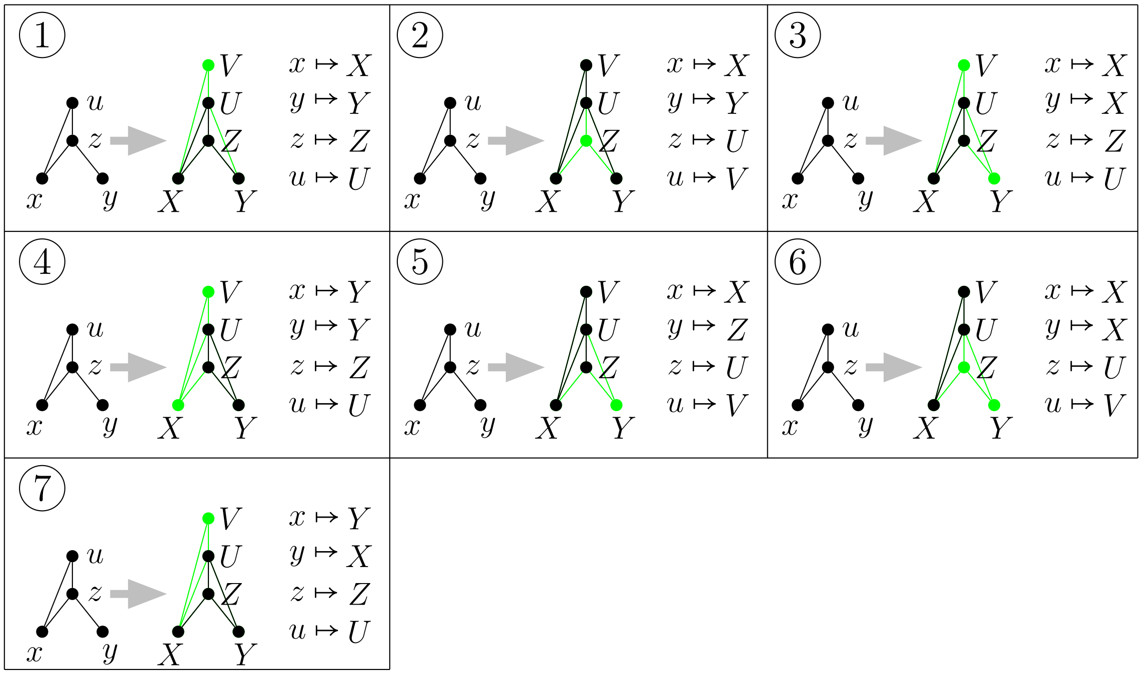
and

$$x_{ij} < x_{i-m, 3^{-m}(j+1)}, x_{i+k, 3^k j+1} \text{ for all } k > 0$$
$$\text{if } v_3(j+1) = m$$

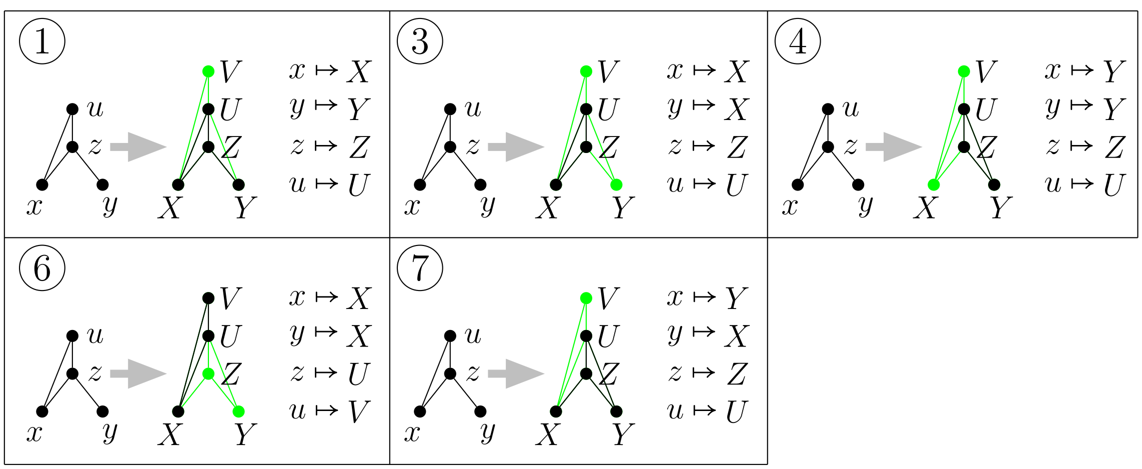
where v_3 is the 3-adic valuation.

BDCST

Morphisms from C_1 to C_2.

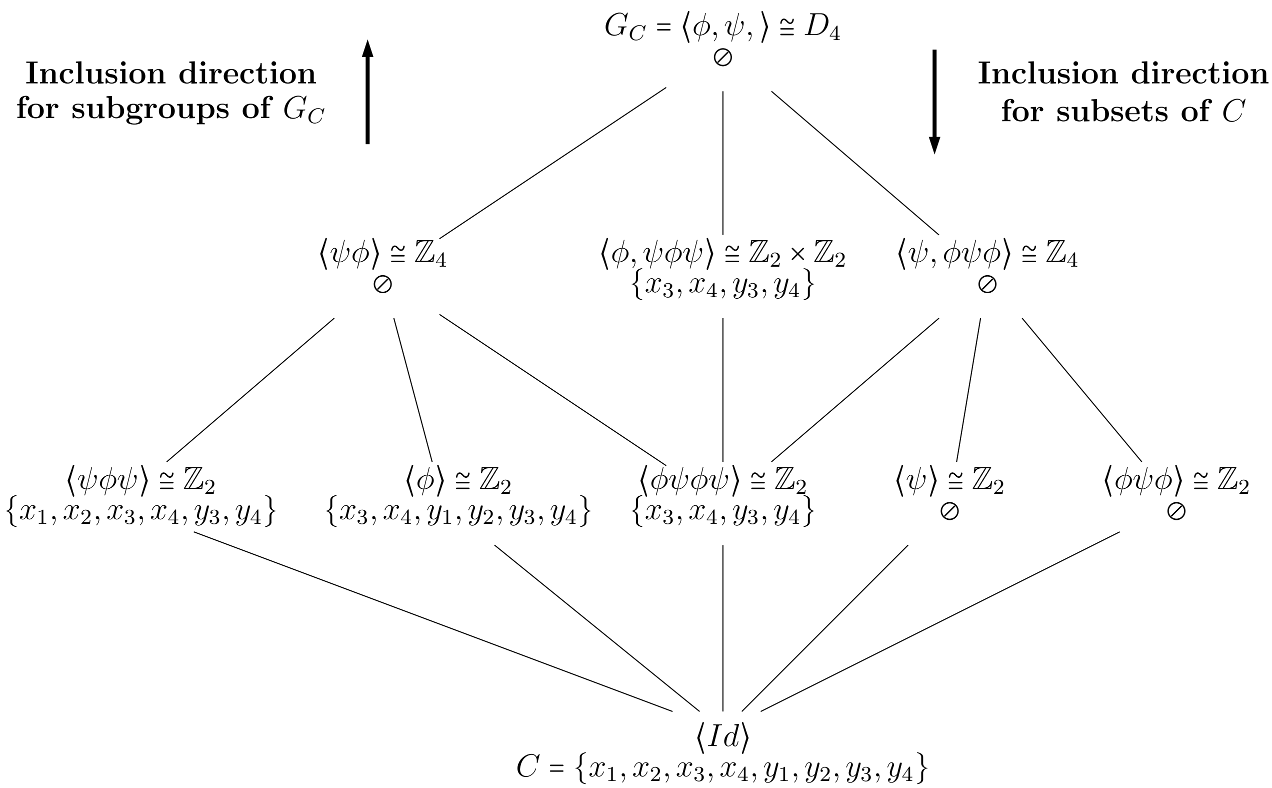


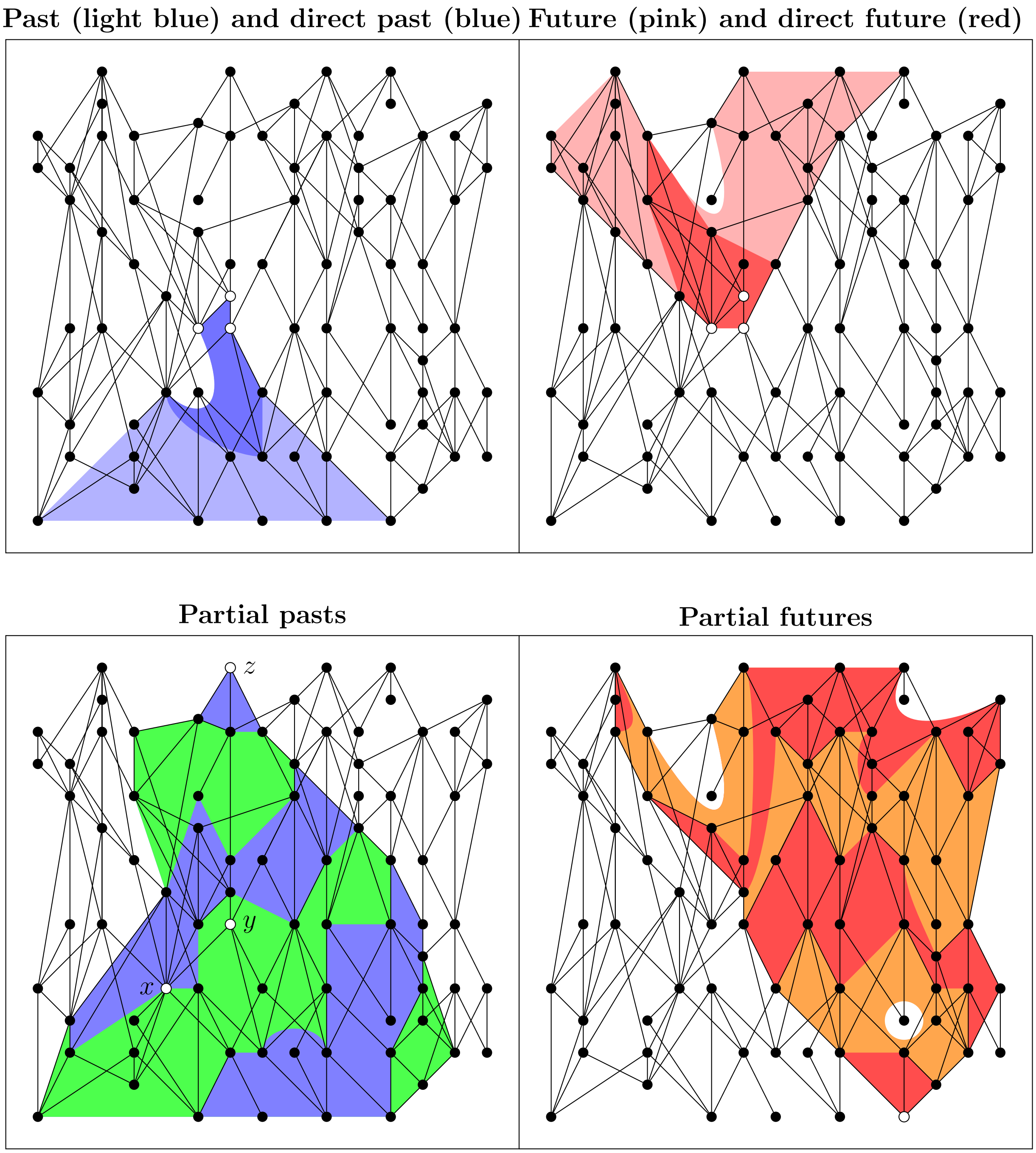
Strong morphisms from C_1 to C_2.

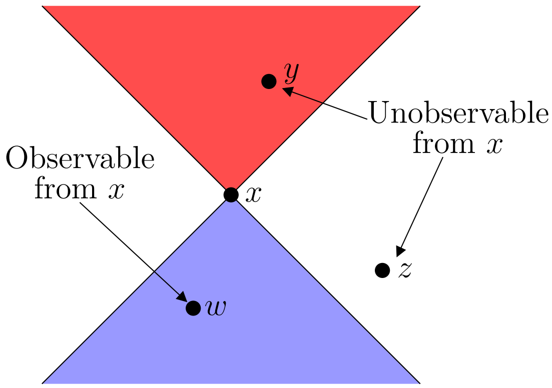


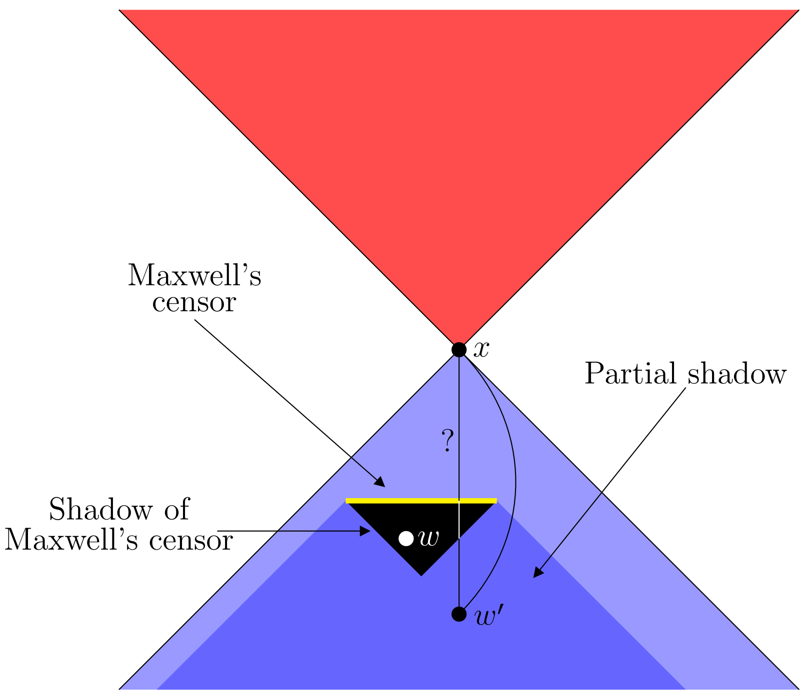
Automorphisms of C.

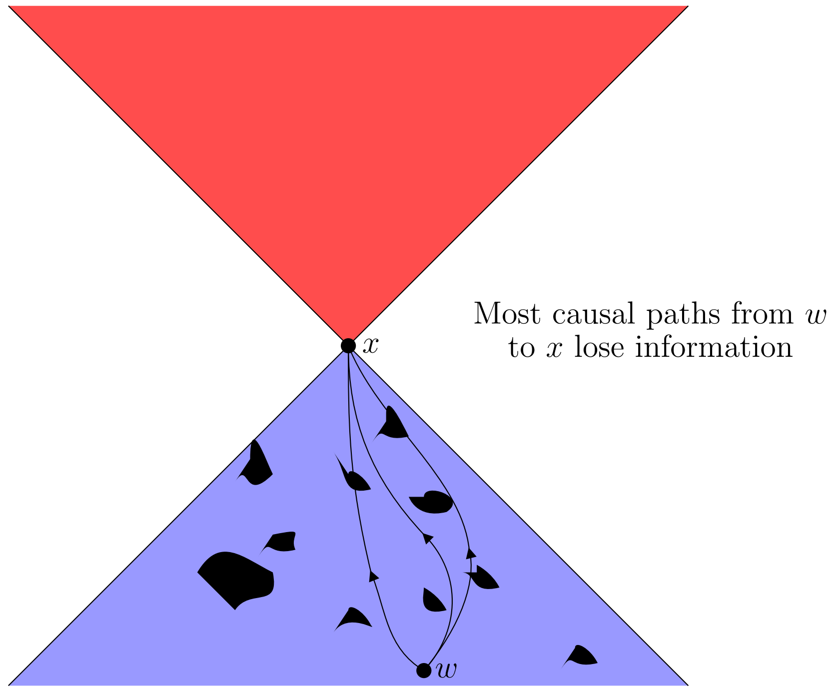
Id (1)	ϕ (2)	$\psi\phi\psi$ (2)	$\phi\psi\phi\psi = \psi\phi\psi\phi$ (2)	ψ (2)	$\psi\phi$ (4)	$\phi\psi$ (4)	$\phi\psi\phi$ (2)
$x_1 \mapsto x_1$	$\mathbf{x_1 \mapsto x_2}$	$x_1 \mapsto x_1$	$\mathbf{x_1 \mapsto x_2}$	$x_1 \mapsto y_1$	$\mathbf{x_1 \mapsto y_2}$	$x_1 \mapsto y_1$	$\mathbf{x_1 \mapsto y_2}$
$x_2 \mapsto x_2$	$\mathbf{x_2 \mapsto x_1}$	$x_2 \mapsto x_2$	$\mathbf{x_2 \mapsto x_1}$	$x_2 \mapsto y_2$	$\mathbf{x_2 \mapsto y_1}$	$x_2 \mapsto y_2$	$\mathbf{x_2 \mapsto y_1}$
$x_3 \mapsto x_3$	$x_3 \mapsto x_3$	$x_3 \mapsto x_3$	$x_3 \mapsto x_3$	$x_3 \mapsto y_3$	$x_3 \mapsto y_3$	$x_3 \mapsto y_3$	$x_3 \mapsto y_3$
$x_4 \mapsto x_4$	$x_4 \mapsto x_4$	$x_4 \mapsto x_4$	$x_4 \mapsto x_4$	$x_4 \mapsto y_4$	$x_4 \mapsto y_4$	$x_4 \mapsto y_4$	$x_4 \mapsto y_4$
$y_1 \mapsto y_1$	$y_1 \mapsto y_1$	$\mathbf{y_1 \mapsto y_2}$	$\mathbf{y_1 \mapsto y_2}$	$y_1 \mapsto x_1$	$y_1 \mapsto x_1$	$\mathbf{y_1 \mapsto x_2}$	$\mathbf{y_1 \mapsto x_2}$
$y_2 \mapsto y_2$	$y_2 \mapsto y_2$	$\mathbf{y_2 \mapsto y_1}$	$\mathbf{y_2 \mapsto y_1}$	$y_2 \mapsto x_2$	$y_2 \mapsto x_2$	$\mathbf{y_2 \mapsto x_1}$	$\mathbf{y_2 \mapsto x_1}$
$y_3 \mapsto y_3$	$y_3 \mapsto y_3$	$y_3 \mapsto y_3$	$y_3 \mapsto y_3$	$y_3 \mapsto x_3$	$y_3 \mapsto x_3$	$y_3 \mapsto x_3$	$y_3 \mapsto x_3$
$y_4 \mapsto y_4$	$y_4 \mapsto y_4$	$y_4 \mapsto y_4$	$y_4 \mapsto y_4$	$y_4 \mapsto x_4$	$y_4 \mapsto x_4$	$y_4 \mapsto x_4$	$y_4 \mapsto x_4$



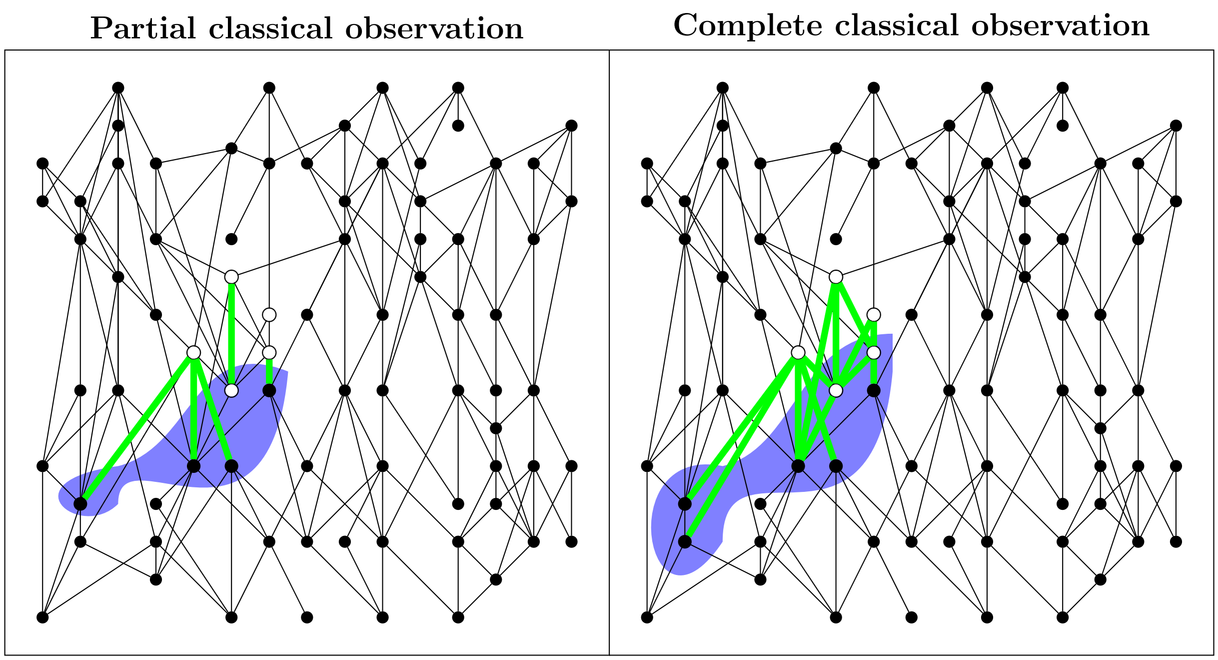

Past (light blue) and direct past (blue)
Future (pink) and direct future (red)
Partial pasts
z
y
x
Partial futures


y
Unobservable from x
Observable from x
x
z
w

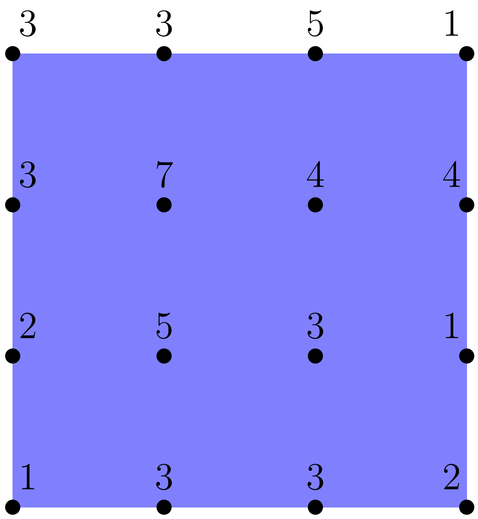

Maxwell's censor
x
Partial shadow
?
Shadow of Maxwell's censor
w
w'

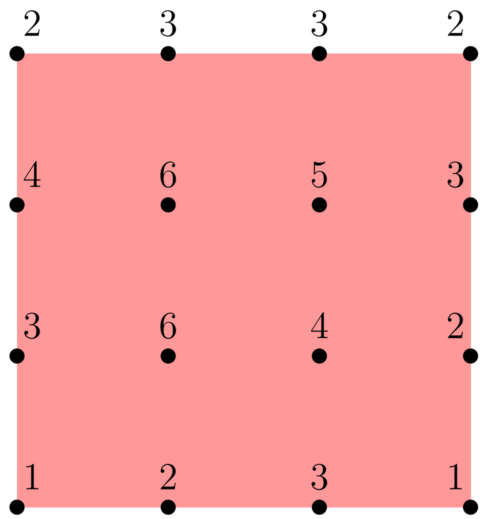


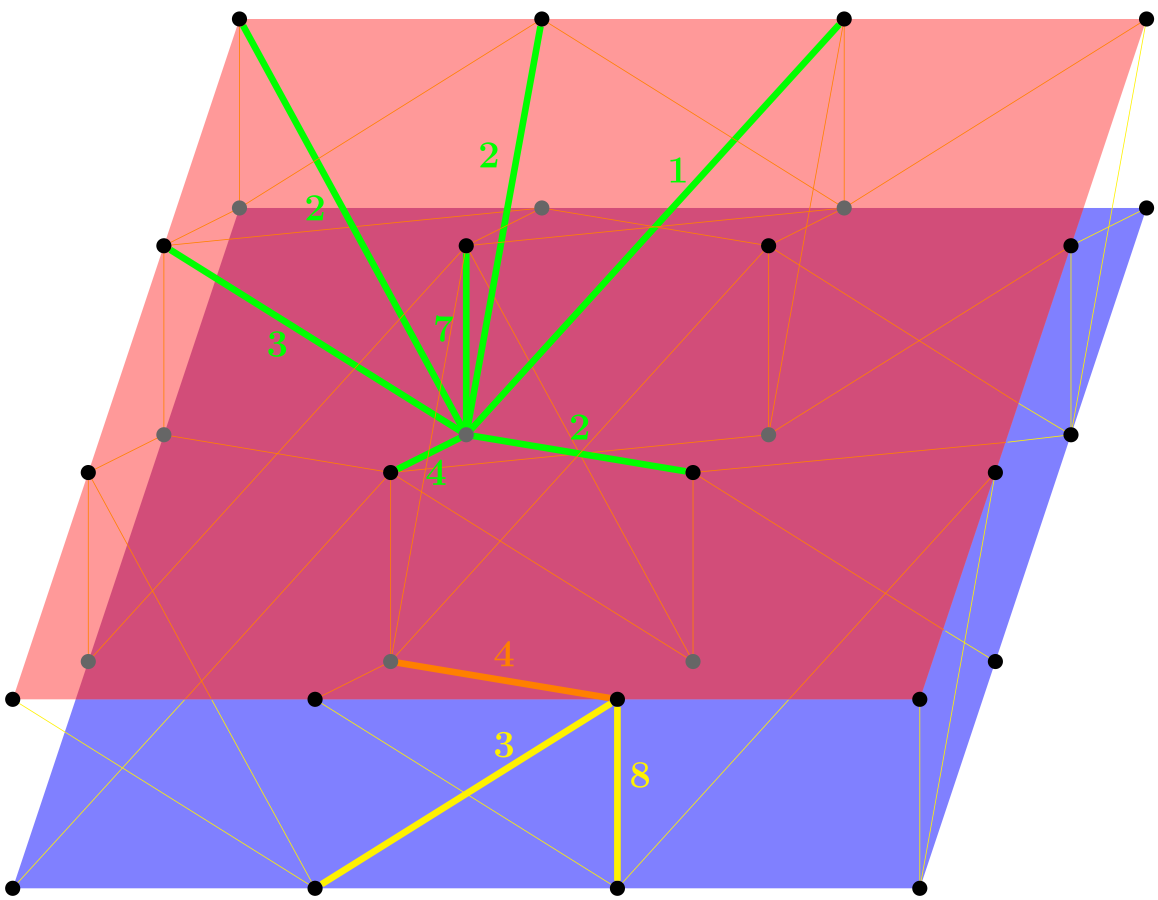
x
w
Most causal paths from w
to x lose information

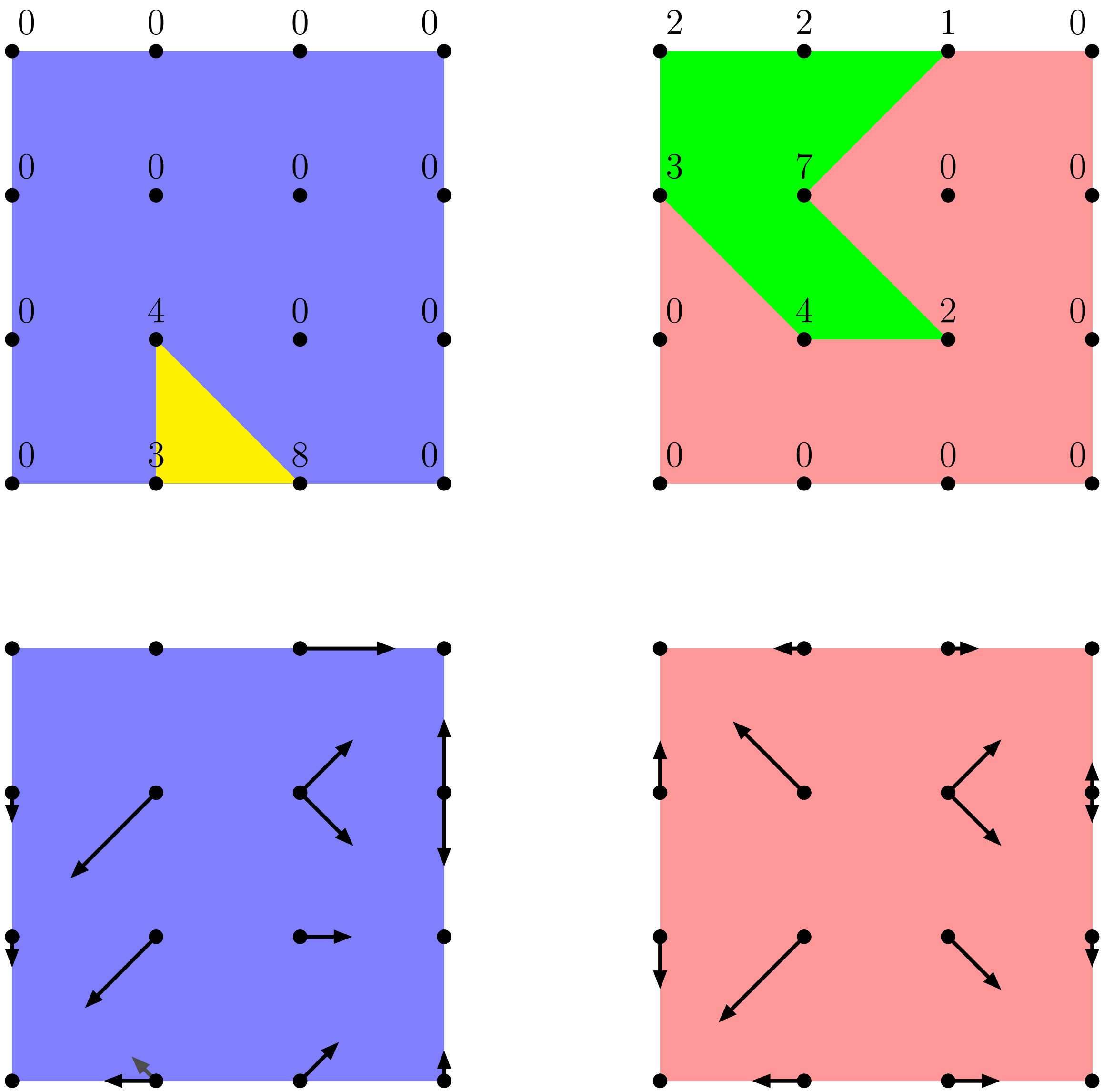


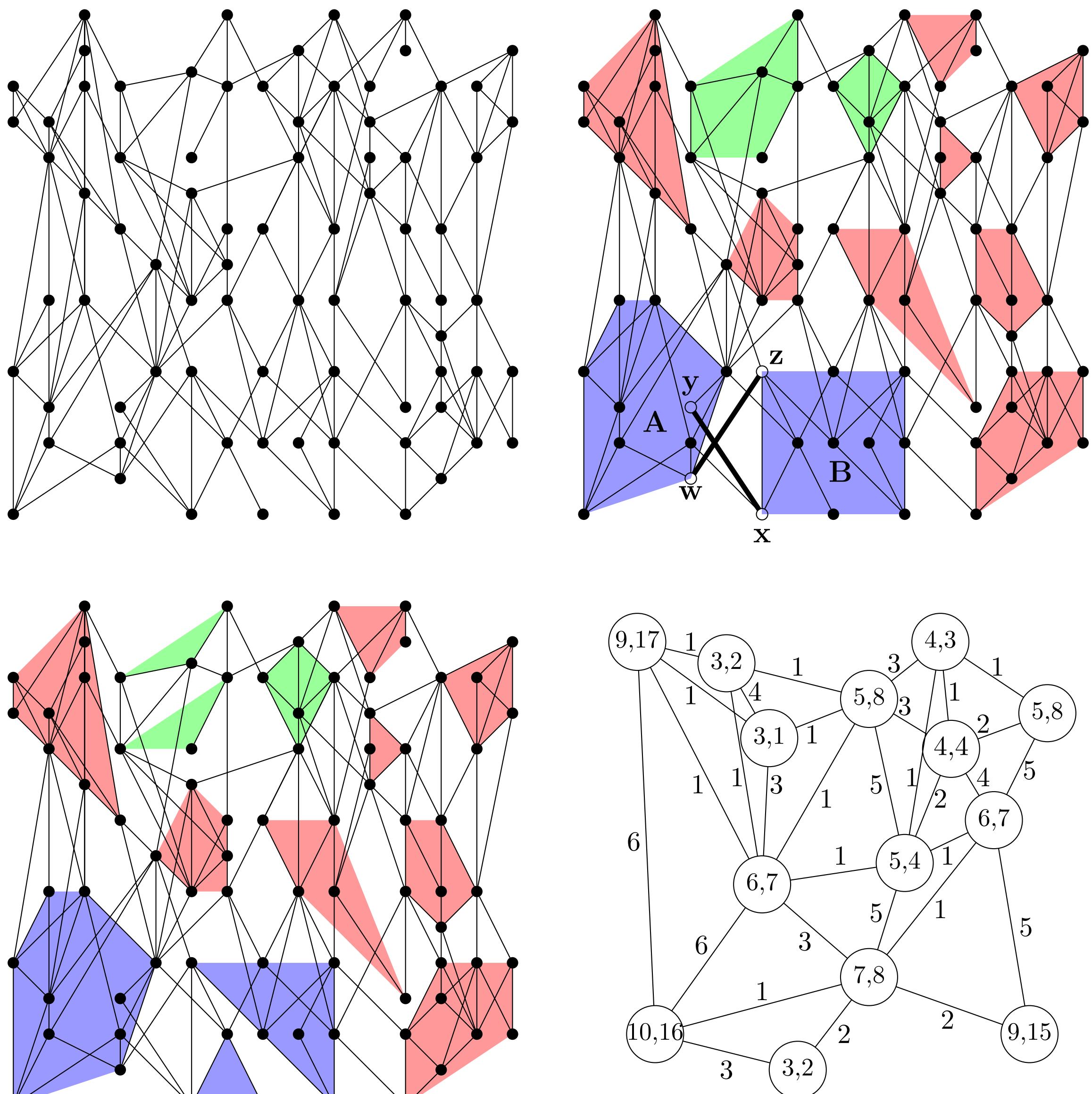
Partial classical observation
Complete classical observation

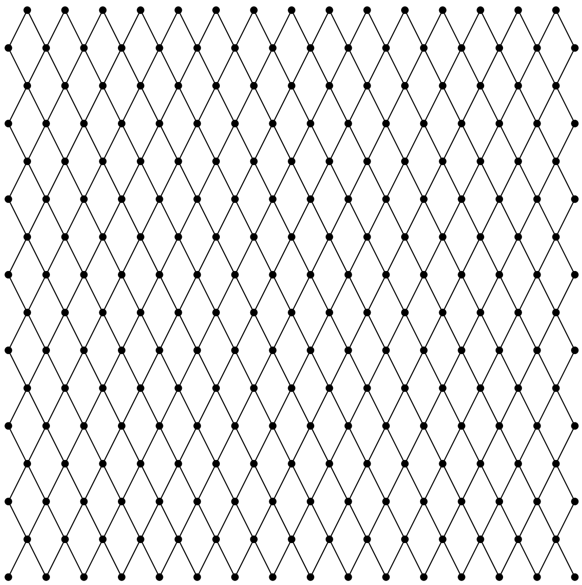

3 3 5 1
3 7 4 4
2 5 3 1
1 3 3 2

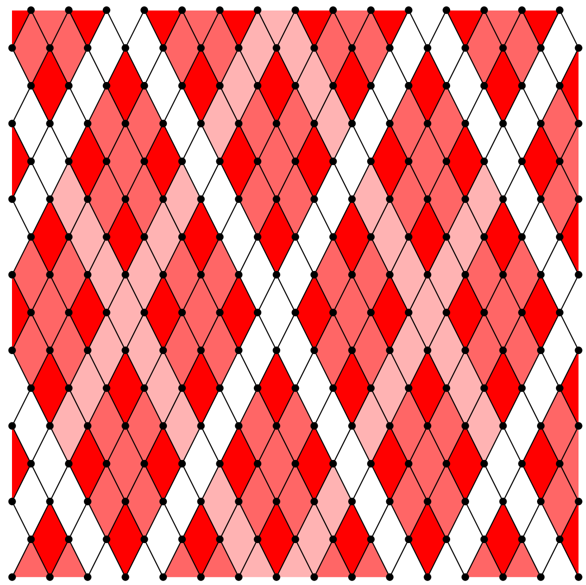

2 3 3 2
4 6 5 3
3 6 4 2
1 2 3 1

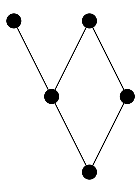

2
1
2
7
3
2
4
4
3
8

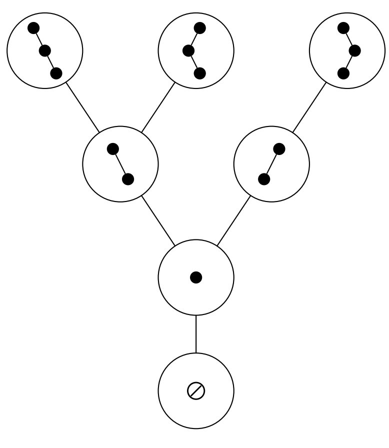

0 0 0 0
0 0 0 0
0 4 0 0
0 3 8 0
2 2 1 0
3 7 0 0
0 4 2 0
0 0 0 0

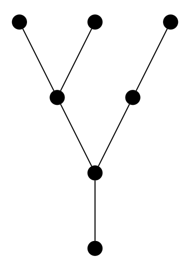

y
z
A
w
x
B
9,17
3,2
4,3
5,8
3,1
4,4
5,8
6,7
5,4
6,7
7,8
10,16
3,2
9,15

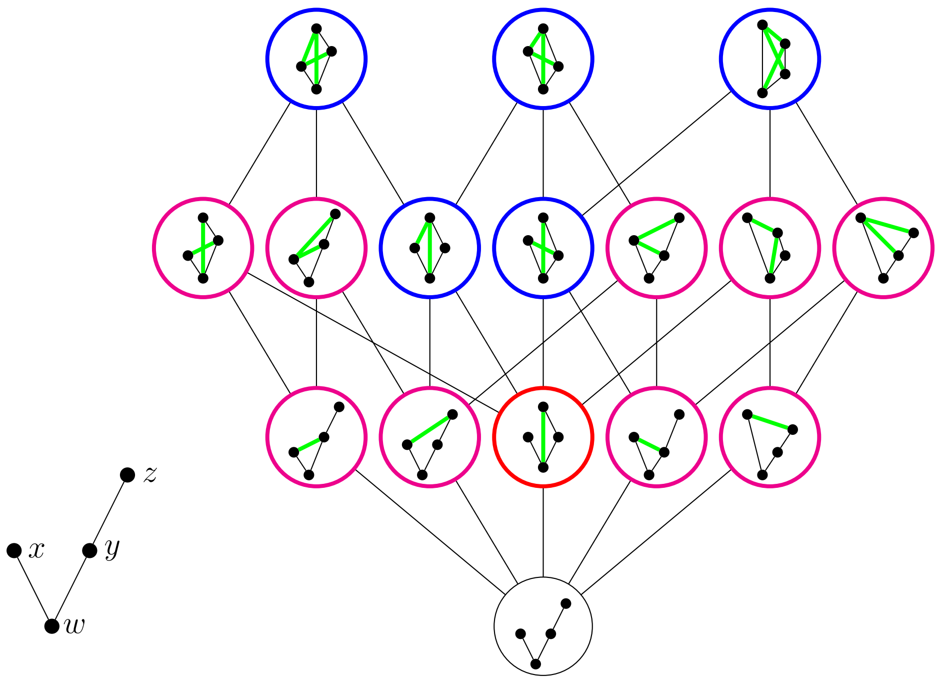




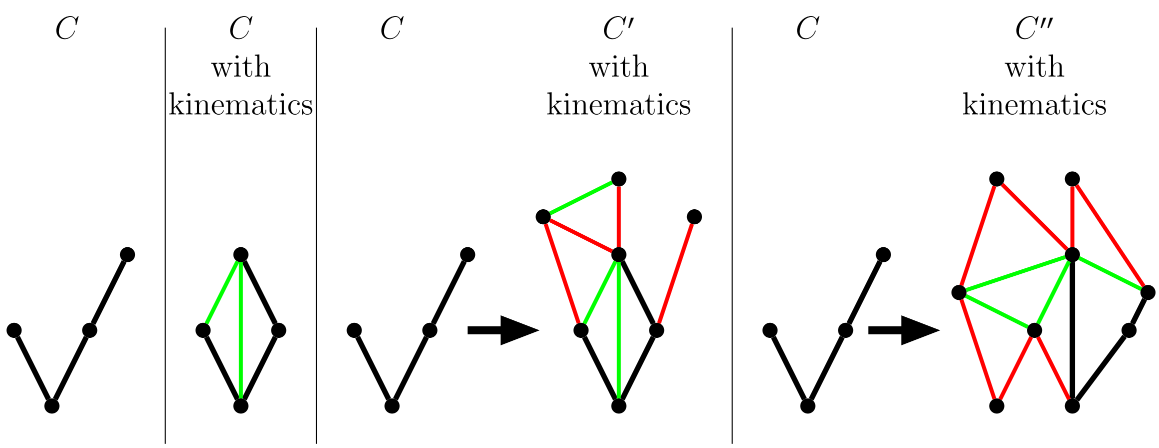




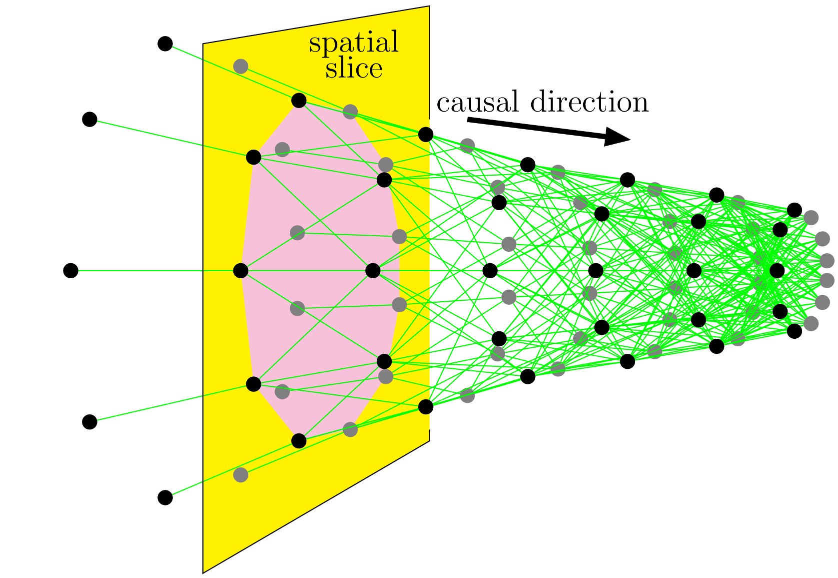


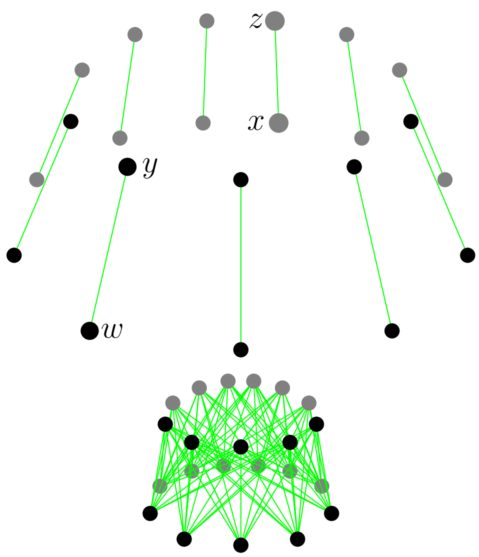


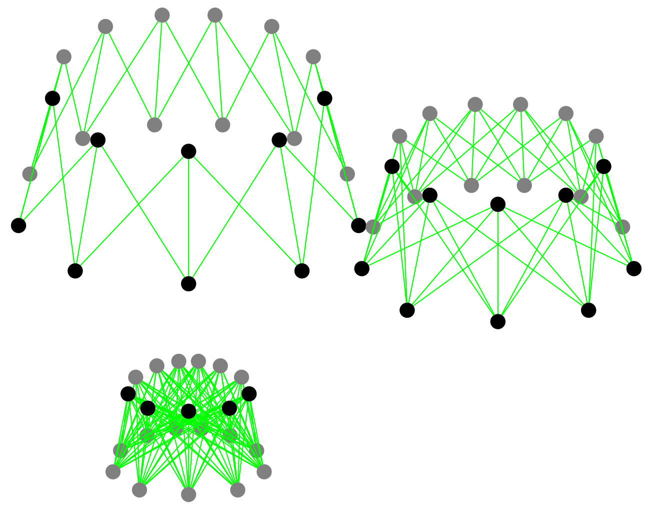
z
x
y
w

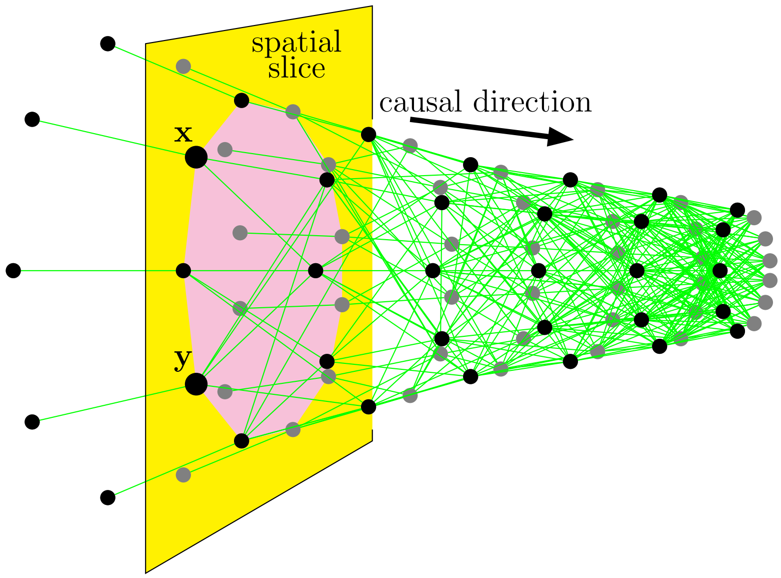


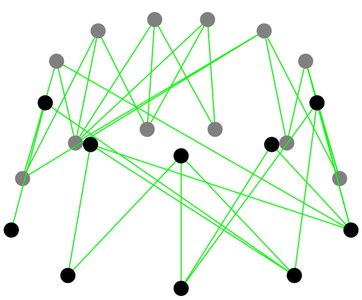
C
C
with
kinematics
C
C′
with
kinematics
C
C″
with
kinematics

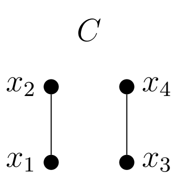

spatial
slice
causal direction

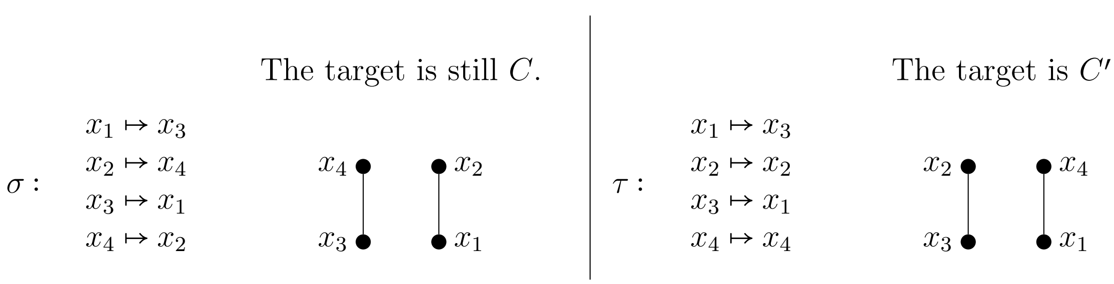

z
x
y
w

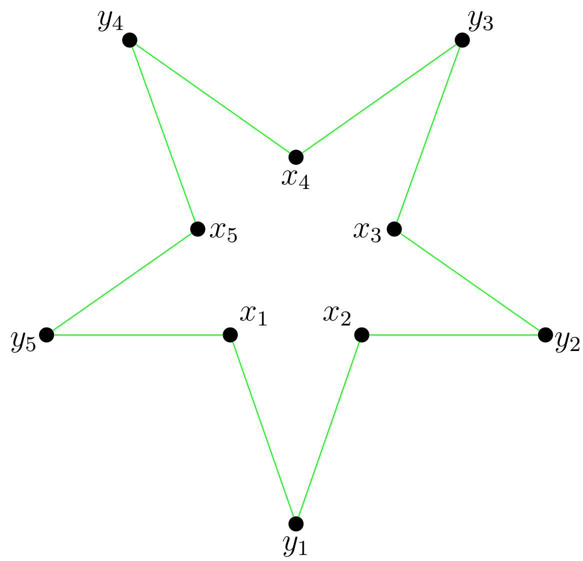



spatial slice
causal direction
x
y




C
x_2
x_4
x_1
x_3


The target is still C.
$\sigma :$
$x_1 \mapsto x_3$
$x_2 \mapsto x_4$
$x_3 \mapsto x_1$
$x_4 \mapsto x_2$
x_4
x_2
x_3
x_1
The target is C'.
$\tau :$
$x_1 \mapsto x_3$
$x_2 \mapsto x_2$
$x_3 \mapsto x_1$
$x_4 \mapsto x_4$
x_2
x_4
x_3
x_1



Five elements.