Bell's Theorem

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Introduction.

- Quantum Theory makes predictions that challenge intuitive notions of physical reality. Einstein and others were sufficiently troubled by these predictions to suggest in the 1930's that quantum theory is incomplete.
- The "missing data" was thought to be described by **hidden variable** theories.
- In 1964, John Bell showed how to experimentally test for hidden variables.
- Experiments based on **Bell's Theorem** seem to rule out hidden variables and support quantum theory.
- Researchers studying the foundations of quantum theory are still trying to understand the implications of Bell's theorem.

Local Realism.

From the dawn of the scientific revolution until the beginning of the twentieth century, most scientists accepted the following two principles of **local realism** without question:

- **Locality:** A physical system is directly influenced only by its immediate environment; distant influences are always mediated by the immediate environment.
- **Counterfactual Definiteness:** It makes sense to talk about properties of a system without measuring them. For example, without measuring the position of a particle, it makes sense to say it has *some* position.

Quantum Theory I.

Beginning in the first years of the twentieth century, and particularly during the 1920's and 1930's, local realism was seriously challenged and eventually almost completely overturned by quantum theory, which is based on the following principles:

- The **state** of a physical system is represented by a vector $|\psi\rangle$ in a **Hilbert space.**
- Observable quantities such as position and momentum are represented by **self-adjoint operators** on the Hilbert space.

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Quantum Theory II.

- If a system is in a state $|\psi\rangle$, then measurement of the observable quantity corresponding to the operator Ω yields one of the **eigenvalues** of Ω. The probability of obtaining a particular value depends on the **projections** of the state vector $|\psi\rangle$ onto the various **eigenspaces** of Ω, and the measurement immediately alters the state to the projection of $|\psi\rangle$ on the eigenspace corresponding to the measured eigenvalue.
- The evolution of the state vector is described by the **Schrödinger equation**:

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$$

where H is the **Hamiltonian**.

Quantum Theory III: Uncertainty Principle.

- Operators representing two observable quantities may not commute (example: **Pauli spin matrices**)
- The **Heisenberg uncertainty principle** states that it is impossible to simultaneously know precise values of two observable quantities corresponding to **noncommuting operators**.

Quantum Theory III: Violation of Local Realism.

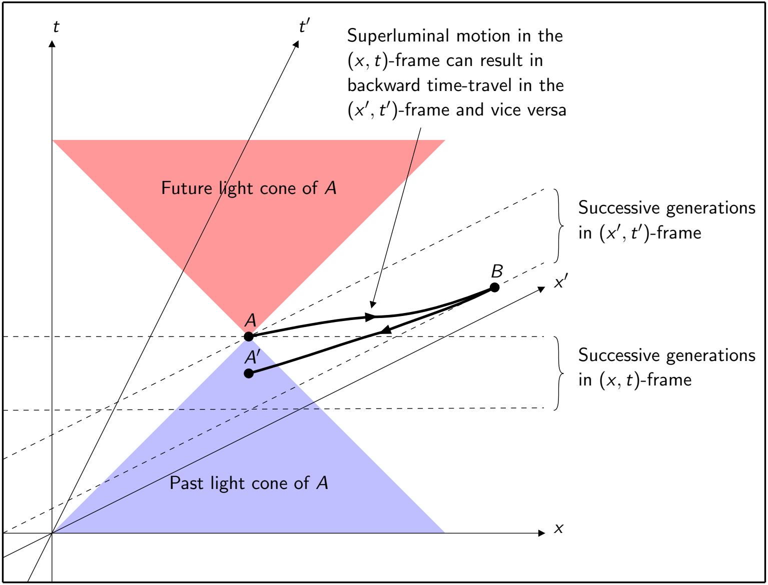
The quantum theory of **entangled states** seems to violate both counterfactual definiteness and locality.

- Counterfactual definiteness is violated because of the uncertainty principle.
- Locality is violated because measurement of one particle instantaneously determines properties of the other.

Why is Violation of Locality a Problem?

- If distant phenomena can directly effect a system without propagation through the intervening spacetime, then influence can travel at arbitrary speeds.
- Einstein's special theory of relativity implies that **superluminal communication**, the transfer of information at speeds faster than the speed of light, can result in **violation of causality**.

Superluminal Communication.



Superluminal motion in the (x, t)-frame can result in backward time-travel in the (x', t')-frame and vice versa

Successive generations in (x', t')-frame

Successive generations in (x, t)-frame

Is Quantum Theory Complete?

At first glance, it seems that a hidden variable theory could rescue both counterfactual definiteness and locality in the quantum theory of entangled states.

- Counterfactual definiteness would be preserved because the hidden variable would contain the “missing information” obscured by the uncertainty principle.
- Locality would be preserved because the existence of well-defined properties (specified by the hidden variable) for each separate part of the entangled state would eliminate the need to hypothesize that the properties of one particle are instantaneously determined by measuring the properties of the other.

The EPR Paper.

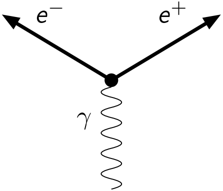
- In 1935, Einstein, Podolsky, and Rosen authored a paper entitled *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?*
- Their conclusion was NO.
- Their argument was based on a thought experiment involving entangled states.

Physical Background I: Entangled Pairs.

Under appropriate circumstances a high-energy photon can convert into an electron and positron. This is symbolized by the equation

$$\gamma \to e^- + e^+$$

where γ is the photon, e^- the electron, and e^+ the positron. This process is called **pair production**. Pair production is often represented by diagrams like the following, called **Feynman diagrams**:



Physical Background II: Spin.

- The electron e^- is a **fermion,** a type of particle with intrinsic angular momentum, called **spin**.
- the measured value $S_l(e^-)$ of the electron spin along an arbitrary axis l always has a value of $\pm\frac{\hbar}{2}$, where $\hbar$ is Planck's reduced constant.
- The positron is the **antiparticle** corresponding to the electron, and thus has the same spin magnitude.
- Photons are **bosons** with zero spin.
- By conservation of angular momentum, the spin measurements of the electron-positron pair along any given axis must be equal and opposite.

Physical Background III: Choice of Coordinates.

Choose coordinates x, y, and z such that the Pauli spin matrices take the form

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Recall that $S_x(e^-)$ means the spin of the electron e^- measured along the x-axis. In this way, the measured values S_x, S_y, and S_z correspond to the operators σ_x, σ_y, and σ_z.

Physical Background III: The Entangled State.

In the x, y, z coordinate system, the entangled state of the electron-positron pair is given by

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|+x\rangle \otimes |-x\rangle - |-x\rangle \otimes |+x\rangle\Big)$$

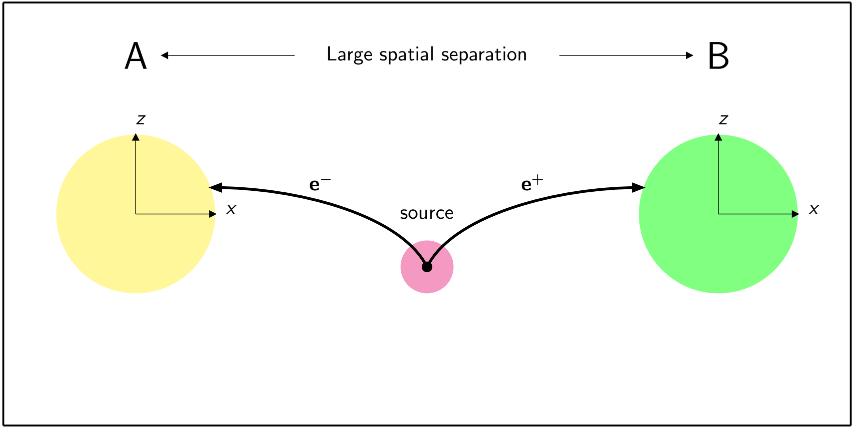
where

$$|+x\rangle \leftrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad |-x\rangle \leftrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

The EPR Experiment I.

- Produce an electron-positron pair in the state $|\psi\rangle$ and separate the two particles by a large distance.
- If $S_x(e^-)$ is measured, this determines $S_x(e^+)$ *without* performing another measurement, by conservation of momentum.
- The same is true for any other axis; in particular, the z-axis.

The EPR Experiment II.



EPR Argument I: Spin Determined at the Source?

Now reason as follows: First suppose that the value of $S_x(e^-)$, and therefore $S_x(e^+)$, is determined at the source, when the particles are first produced. If now $S_z(e^-)$ is measured instead of $S_x(e^-)$, this yields a definite value of $S_z(e^+)$. There are two possibilities:

1. The positron e^+ has precise simultaneous values of S_x and S_z. This violates the uncertainty principle.
2. Measuring $S_z(e^-)$, thus obtaining $S_z(e^+)$, instantaneously affects the spin of e^+ along the x-axis in such a way that e^+ no longer has a precise value of S_x. This violates locality.

EPR Argument II: Spin Determined by Measurement?

Suppose instead that the value of $S_x(e^-)$, and therefore $S_x(e^-)$, is only determined when $S_x(e^-)$ is actually measured. This immediately violates counterfactual definiteness. Further, the measurement of $S_x(e^-)$ instantaneously affects the spin of e^+ along the x-axis in such a way that the two spins cancel, so locality is violated as well.

Uncertainty Principle or Local Realism?

- One way or the other, either the uncertainty principle or local realism is violated.
- Einstein, Podolsky, and Rosen chose to believe local realism. They concluded that their argument proves the incompleteness of quantum mechanics.
- In this view, the uncertainty principle is a shortcoming of quantum theory, not a fundamental principle of nature.

Bell's Theorem I: Hidden Variables Detectable!

In a 1964 paper, John Bell showed that the presence of hidden variables has experimental consequences.

- Bell's result applies to hidden variables affecting an entangled system at the source of entanglement.
- For simplicity, we will consider the case where the entangled system is an electron-positron pair in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|+x\rangle \otimes |-x\rangle - |-x\rangle \otimes |+x\rangle\Big)$$

- Bell's result is a statistical one, so we need to produce many copies of this state (note that this is allowed!).

Setup for Bell's Theorem.

Imagine that two distant observers, A and B, make measurements on the copies of $|\psi\rangle$.

- Observer A measures e^-. She uses one of two detector settings, either a or a'. For example, a and a' might be spin measurements along different axes.
- Observer B measures e^+. He uses one of two detector settings, either b or b'. For example, b and b' might be spin measurements along different axes.

Adding Hidden Variables.

To include the possibility of hidden variables, we explicitly take the measurements to depend on an additional variable λ as well as the detector settings.

- λ is the hidden variable. It affects the entangled pair at the source.
- We denote by $A(a, \lambda)$ the value measured by A with detector setting a and hidden variable value λ.
- Similarly for $A(a', \lambda)$, $B(b, \lambda)$, and $B(b', \lambda)$.
- The value of λ may differ for different pairs. Formally, λ belongs to a probability space Λ with probability measure $\rho(\lambda)$ giving the probability that a representative pair has hidden variable value λ.

Bell's Theorem II: Bell's Inequality.

Bell proved an inequality about the correlations among the measurements performed by A and B, assuming the presence of a hidden variable. The form of **Bell's inequality** we will use is

$$-2 \leq C \leq 2$$

where, C, called the **correlation**, is given by

$$C := C(A(a), B(b)) - C(A(a), B(b')) + C(A(a'), B(b)) + C(A(a'), B(b'))$$

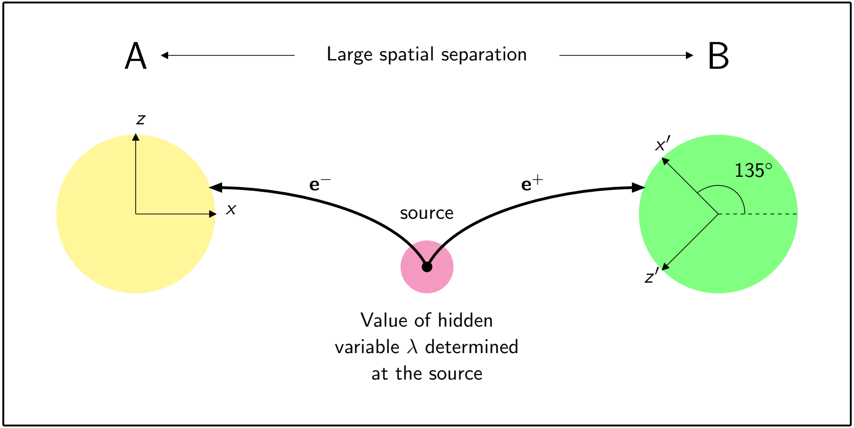
The hidden variable λ does not appear in this formula because the correlations are obtained by integrating over the probability space Λ. This is as it should be, since A and B cannot measure λ.

Quantum Theory Contradicts Bell's Inequality.

Bell then specified a particular type of experiment for which quantum theory predicts a violation of Bell's inequality. We will examine the following experimental version:

- The detector setting a represents the measurement of the electron spin $S_z(e^-)$.
- The detector setting a' represents the measurement of the electron spin $S_x(e^-)$.
- The detector settings b and b' represent spin measurements along axes rotated by 135° with respect to the x, z coordinates.

Bell's Experimental Setup.



Calculating the Quantum Correlations I.

Next, we compute the values of the correlations among the measurements $A(a)$, $B(b)$, etc. predicted by quantum theory. A straightforward though slightly tedious calculation shows that

$$C(A(a), B(b)) = \langle A(a) B(b) \rangle$$

$$= \Big\langle \sigma_z \otimes I \circ \Big(-\frac{1}{\sqrt{2}} I \otimes (\sigma_x + \sigma_z) \Big) |\psi\rangle \Big| |\psi\rangle \Big\rangle$$

$$= \frac{1}{\sqrt{2}}$$

Calculating the Quantum Correlations II.

Similarly,

$$C(A(a',\lambda),B(b,\lambda)) = C(A(a',\lambda),B(b',\lambda)) = \frac{1}{\sqrt{2}}$$

and

$$C(A(a,\lambda),B(b',\lambda)) = -\frac{1}{\sqrt{2}}$$

Thus, the total correlation is

$$C = \mathbf{2\sqrt{2}}$$

This violates Bell's inequality.

Experiments Seem to Support Quantum Theory.

- Bell's Theorem not only showed that a wide class of hidden variable theories disagree with quantum theory rather than merely supplementing it, but also indicated how to test which idea is right experimentally.
- Experimental tests involving optics (different than the experiment described here) have repeated "seemed" to show violation of Bell's inequality.
- At face value, this supports quantum theory and implies that local realism is false.
- However, not everyone is convinced.

Experimental Objections.

- To date, experimental tests of Bell's theorem are not as clean and straightforward as the hypothetical experiment involving entangled pairs.
- Interpretation of optical tests of Bell's theorem involve particular assumptions about the electromagnetic field that are generally believed to be true but are not essential to quantum theory.
- If these assumptions are false, existing experimental results could agree with both local realism and quantum theory.

Possible Interpretations I.

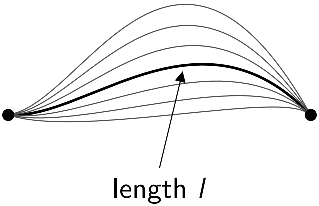
- The most popular and mainstream interpretation of Bell's Theorem and the associated experimental results is that quantum theory is true and local realism is false.
- As stated above, some scientists are not prepared to accept the experimental results because of possible experimental flaws or extra assumptions.
- Others have advocated **nonlocal hidden variable theories**. In such theories, counterfactual definiteness holds, but particles can exchange information at arbitrary speeds to produce the necessary correlations.

Possible Interpretations II.

- Another idea is that the histories of the particles do not agree “at first,” but the history is corrected (1984-style) before we can tell the difference.
- Superdeterminism would eliminate the need for instantaneous communication by simply specifying every detail of every event at the outset.
- Finally, it is possible that quantum “nonlocality” reflects small-scale **non-manifold structure of spacetime**. Let us explore this idea a little further.

A Property of Manifolds.

Manifolds have the following obvious but nontrivial property: if there is one path between two points of a given length l, there are many other paths of length close to l between the two points.



What does Locality Really Mean?

- Consider again the locality principle as stated above: **A physical system is directly influenced only by its immediate environment.**
- For this statement to have any meaning, we must be able to define the "immediate environment" *independently* of actual influences among systems.
- In other words, we are assuming that the "spacetime manifold" has a metric structure *independent* of the causal structure of the universe.
- But "spacetime" may not even be a manifold on small scales! In this case, the principle of locality loses its meaning.

An Alternative View of Locality I.

Consider the following alternative to the locality principle: **The local environment of a system is the region of the universe that directly interacts with the system.**

- This viewpoint turns the discussion on its head by redefining locality. It takes the causal structure to be fundamental, rather than the metric structure.
- In classical physics (general relativity), this definition is indistinguishable from the locality principle, since the causal and metric structures agree (we think!).

An Alternative View of Locality II.

- With this definition, entangled systems are *always* local, even though there may be only one ”short path” between them, with every other path being “long.”
- Since no metric space has these properties, the phenomenon of entanglement is viewed as evidence of the non-manifold structure of spacetime at small scales.

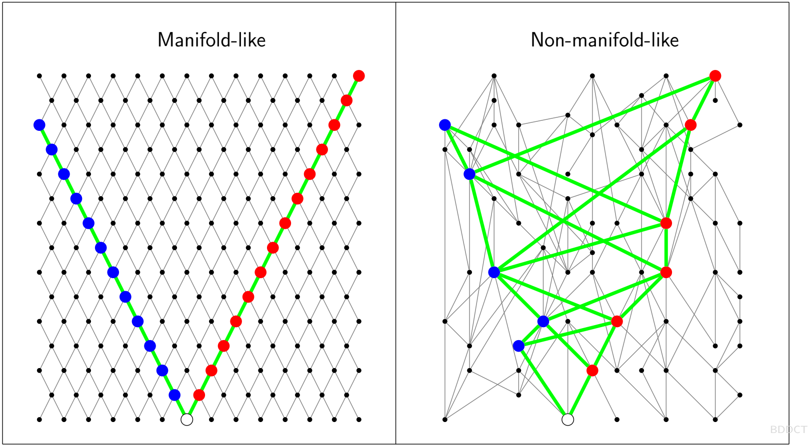
Other Hints of Non-manifold Structure: Quantum Gravity.

- In quantum field theory, continuous fields are quantized to produce discrete particles.
- General Relativity describes the metric structure of spacetime itself as a continuous tensor field, but it is hard to quantize (if it is even correct).
- Presumably, a quantum theory of gravity would involve **discrete units of spacetime**; i.e. a non-manifold structure at small scales.
- Attempts in this direction include **string theory**, **loop quantum gravity**, **causal dynamical triangulation**, and more generally, **discrete causal theory**.

My Pet Idea: Discrete Causal Theory I.

- Discrete causal theory may be able to reconcile Bell's theorem and the accompanying experiment results with an appropriate version of locality.
- In particular, this would eliminate the troubling causal consequences of nonlocality.
- The following slide shows a model of an entangled system in a non-manifold-like discrete causal structure.

My Pet Idea: Discrete Causal Theory II.



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